Probability

ISC Class 12 Board Questions

With Answers

1993 to 2022

1. The probabilities a student getting I, II, III division in an examination are respectively $\frac{1}{10}, \frac{3}{5} and \frac{1}{4}$. Calculate the probability that the student fails in the examination. (ISC 1993)

2. Three shots are fired at a moving target. The probabilities of hitting the target at the first, second and third shots are $\frac{1}{2}, \frac{3}{10} and \frac{1}{10}$ respectively. Calculate the probability that the target gets hit. (1994)

3. **A** speaks truth in 55% cases and B speaks truth in 75% cases. Determine the percentage of cases in which they are likely to contradict each other in stating the same fact.

(ISC 1995)

4. Find the probability of drawing an ace or spade or both from a deck of cards. (ISC 1996)

5. A problem in mathematics is given to three students **A**, **B** and **C**. The chances of solving it by **A**, **B** and **C** are respectively $\frac{1}{3}, \frac{1}{4} and \frac{1}{5}$.Calculate the probability that the problem that the problem will be solved. (ISC 1997,2003)

6. In a given race, the odds are in favour of four horses A, **B**, **C**, **D** are 1:3, 1:4, 1:5, 1:6 respectively. Assuming that a dead heat is impossible, find the chance that one of them wins the race.

(ISC 1998)

7. The probability that a boy will not pass MBA examination is $\frac{3}{5}$ and that a girl will not pass is $\frac{4}{5}$ .Calculate the probability that at least one of them passes the examination.

(ISC 1999)

8. A card is drawn at random from a pack of 52 playing cards. What is the probability that the card drawn is neither a spade or a queen?

(ISC 2006, 2000)

9. A bag contains 20 balls marked 1 to 20. One ball is drawn at random from the bag. What is the probability that the ball drawn is marked with a number which is a multiple of 5 or 7.

(ISC 2001)

10. A problem in mathematics is given to four students A, B, C and D. Their chances of solving the problems are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}\ and\ \frac{1}{5}$ respectively. What is the probability that the problem will be solved? (ISC 2001)

11. A pair of dice is thrown. If two numbers appearing on them is different, find the probability that the sum of the numbers appearing is 6.

(ISC 2001)

12. There are 10 persons who are to be seated together around a circular table. Find the probability that the two particular persons will always seat together.

(ISC 2002)

13. One number is chosen at random from the numbers 1 to 21. Find the probability that it may be a prime number.

(ISC 2003)

14. A bag has 4 red and 5 black, a second bag has 3 red and 7 black balls. One ball is drawn from 1^{st} bag and two balls from 2^{nd} bag. Find the probability that two balls are black, and one is red.

(ISC 2004)

15. A candidate is selected for interview of management trainees in 3 companies. For the first company, there are 12 candidates, for the second there are 15 candidates and for the third, there are 10 candidates. Find the probability that he is selected by at least one of the companies. (ISC 2005)

16. The probability of A, B and C solving a problem are $\frac{1}{3}, \frac{2}{7}, \frac{3}{8}$ respectively. If all try and solve the problem simultaneously, find the probability that only one of them solve it. (ISC 2006)

17. **A** and B throw two dice each. If **A** gets a sum of 9 on his two dice, then find the probability of B getting a higher sum.

(ISC 2006)

18. Kamal and Monica appear for an interview for two vacancies. The probability of Kamal's selection is $\frac{1}{3}$ and that of Monica's selection is $\frac{1}{5}$. Find the probability that only one of them will be selected.

(ISC 2007)

19. The bag A contains 3 white and 2 black balls while the bag B contains 2 white and 5 black balls. One of the bags is chosen at random and a ball is drawn from it. What is the probability that the ball is white? (ISC 2007)

20. In a single throw of two dice find the probability of getting a total of at most 9.

(ISC 2008)

21. There are three urns A, B and C. Urn A contains 4 red balls and 3 black balls. Urn B contains 5 red balls and 4 black balls. Urn C consists of 4 red balls and 4 black balls. One ball is drawn from each of these urns. What is the probability that the three balls drawn consist of 2 red balls and 1 black ball?

(ISC 2008)

22. Bag A contains 5 white 4 black balls, bag B contains7 white and 6 black balls. One ball is drawn from bag A and without noticing its colour, is put in the bag B. If the ball is then drawn from bag B, find the probability that it is black in colour.

(ISC 2009)

23. An article manufactured by a company consists of two parts A and B. In this process of manufacture of part A, 9 out of 104 parts may be defective. Similarly, 5 out of 100 are likely to be defective in the manufacture of part B. Calculate the probability that the article manufactured will not be defective.

(ISC 2009)

24. Two horses are considered for a race. The probability of selection of first horse is $\frac{1}{4}$ and that of the second is $\frac{1}{3}$. What is the probability that :

(i) both of them will be selected.

(ii) only one of them will be selected. (ISC 2009)

(iii) none of them will be selected.

25. A fair die is thrown once. What is the probability that either an even number or a number greater than three will turn up? (ISC 2010)

26. Akhil and Ajay appear for an interview for two vacancies. The probability of Akhil's selection is $\frac{1}{4}$ and Ajay's selection is $\frac{2}{3}$. Find the probability that the only one of them will be selected.

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(ISC 2010)

27. There are two bags. One bag contains six green and three red balls. The second bag contains five green and four red balls. One ball is transferred from the first bag to the second bag. Then one ball is drawn from the second bag. Find the probability that it is a red ball.

(ISC 2010)

28. A purse contains 4 silver and 5 copper coins. A second purse contains 3 silver and 7 copper coins. If a coin is taken out at random from one of the purses, what is the probability that it is a copper coin?

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(ISC 2011)

29. Aman and Bhuvan throw a pair of dice alternately. In order to win, they have to get a sum of 8. Find their respective probabilities of winning if Aman starts the game. (ISC 2011)

30. A bag contains 8 red and 5 white balls. Two successive draws of 3 bulbs are made at random from the bag without replacements. Find the probability that the first draw yields 3 white balls and the second draw 3 red balls. (ISC 2012)

31. Bag A contains three red and four white balls; Bag B contains two red and three white balls. If one ball is drawn from bag A and two balls from bag B, find the probability that:
    (i) one ball is red, and two balls are white.
    (ii) all the three balls are of same colour. (ISC 2013)

32. Three persons Aman, Bipin and Mohan attempt a mathematics problem independently. The odds in favour of Aman and Mohon solving the problem are 3 : 2 and 4 : 1 respectively and the odds against Bipin solving the problem are 2 : 1. Find:
    (i) the probability that all the three will solve the problem.
    (ii) the probability that problem will be solved. (ISC 2013)

33. In a college 70% students pass in Physics, 75% pass in mathematics and 10% students fail in both. One student is chosen at random. What is the probability that:
    (i) he passes in Physics and Mathematics.
    (ii) he passes in Mathematics given that he passes in Physics.
    (iii) he passes in Physics given that he passes in Mathematics. (ISC 2014)

34. A bag contains 5 white and 4 black balls, and another bag contains 7 white and 9 black balls. A ball is drawn from the first bag and two balls are drawn from the second bag. What is the probability of drawing one white and two black balls? (ISC 2014)

35. An urn contains 2 white and 2 black balls. A ball is drawn at random. If it is white, it is not replaced into the urn. Otherwise, it is replaced with another ball of the same colour. The process is repeated. Find the probability that the third ball drawn is black. (ISC 2015)

36. Three persons A, B and C shoot to hit a target. If A hits the target four times in five trials, B hits it three times in four trials and C hits it in two times in three trials, find the probability that:
    (i) exactly two persons hit the target.
    (ii) at least two persons hit the target.
    (iii) none hit the target. (ISC 2015)

37. A committee of 4 persons has to be chosen from 8 boys and 6 girls, consisting of at least one girl. Find the probability that the committee consists of more girls than boys. (ISC 2016)

38. An urn contains 10 white and 3 black balls while another urn contains 3 white and 5 black balls. Two balls are drawn from the first urn and put into the second urn and then a ball is drawn from the second urn. Find the probability that the ball drawn from the second urn is a white ball. (ISC 2016)

39. A problem is given to three students whose chances of solving it are $\frac{1}{4}, \frac{1}{5}$, and $\frac{1}{3}$ respectively. Find the probability that the problem is solved. (ISC 2017)

40. In a class of 60 students, 30 opted for mathematics, 32 opted for Biology and 24 opted for both mathematics and Biology. If one of these students is selected at random, find the probability that:
    (i) the student opted for mathematics or Biology.
    (ii) the student has opted neither mathematics nor biology.
    (iii) the student has opted Mathematics but not biology. (ISC 2017)

41. If A and B are events such that P(A) = $\frac{1}{2}$, P(B) = $\frac{1}{3}$ and P(A∩B) = $\frac{1}{4}$, then find:
    (a) P(A/B)
    (b) P(B/A) (ISC 2018)

42. In a race, the probabilities of A and B winning the race are $\frac{1}{3}$ and $\frac{1}{6}$ respectively. Find the probability of neither of them winning the race. (ISC 2018)

43. A speaks the truth in 60% of the cases, while B in 40% of the cases. In what percent of cases are they likely to contradict each other in stating the same fact? (ISC 2018)

44. Two balls are drawn from an urn containing 3 white, 5 red and 2 black balls, one by one without replacement. What is the probability that at least one ball is red? (ISC 2019)

45. If events A and B are independent, such that P(A) = $\frac{3}{5}$, P(B) = $\frac{2}{3}$, find P(AU B). (ISC 2019)

46. Bag A contains 4 white balls and 3 black balls, while Bag B contains 3 white balls and 5 black balls. Two balls are drawn from Bag A and placed in Bag B. Then what is the probability of drawing a white ball from Bag B? (ISC 2019)

47. A bag contains 5 white, 7 red and 4 black balls. If four balls are drawn one by one with replacement, what is the probability that none is white? (ISC 2020)

48. Let A and B be two events such that
    $P(A) = \frac{1}{2}, P(B) = p$ and $P(A \cup B) = \frac{3}{5}$
    Find $'p'$ if A and B are independent events. (ISC 2020)

49. Three persons A, B and C shoot to hit a target. Their probabilities of hitting the target are $\frac{5}{6}, \frac{4}{5}$ and $\frac{3}{4}$ respectively. Find the probability that:
    (i) exactly two persons hit the target.
    (ii) at least one person hit the target. (ISC 2020)

50. If A and B are two events such that P(A) $= \frac{4}{5}$ and P(B/A) $= \frac{7}{8}$ then P (A $\cap$ B) is equal to:

(a) $\frac{7}{40}$

(b) $\frac{21}{40}$

(c) $\frac{32}{35}$

(d) $\frac{7}{10}$ (ISC Sem-2 2022)

51. A bag contains 9 red, 7 white and 4 black balls. If two balls are drawn at random without replacement, the probability that both balls are red will be:

(a) $\frac{11}{95}$

(b) $\frac{18}{95}$

(c) $\frac{18}{85}$

(d) $\frac{18}{23}$ (ISC Sem-2 2022)

52. Two horses are considered for race. The probability of selection of first horse is $\frac{1}{5}$ and that of second is $\frac{2}{3}$. Find the probability that:

(i) both will be selected

(ii) only one of them will be selected

(iii) none of them will be selected

(iv) at least one of them will be selected. (ISC Sem-2 2022)

Answer

(1)	$\frac{1}{20}$	(2)	$\frac{137}{200}$	(3)	47.5%	(4)	$\frac{4}{13}$	(5)	$\frac{3}{5}$
(6)	$\frac{57}{140}$	(7)	$\frac{13}{25}$	(8)	$\frac{9}{13}$	(9)	$\frac{3}{10}$	(10)	$\frac{4}{5}$
(11)	$\frac{1}{9}$	(12)	$\frac{2}{9}$	(13)	$\frac{8}{21}$	(14)	$\frac{7}{15}$	(15)	$\frac{23}{100}$
(16)	$\frac{25}{56}$	(17)	$\frac{1}{6}$	(18)	$\frac{2}{5}$	(19)	$\frac{31}{70}$	(20)	$\frac{5}{6}$
(21)	$\frac{17}{42}$	(22)	$\frac{29}{63}$	(23)	$\frac{361}{416}$	24(i)	$\frac{1}{12}$	24(ii)	$\frac{5}{12}$
24(iii)	$\frac{1}{2}$	(25)	$\frac{2}{3}$	(26)	$\frac{7}{12}$	(27)	$\frac{13}{30}$	(28)	$\frac{113}{180}$
(29)	0.463	(30)	$\frac{7}{429}$	31(i)	$\frac{33}{70}$	31(ii)	$\frac{3}{14}$	32(i)	$\frac{4}{25}$
32(ii)	$\frac{71}{75}$	33(i)	$\frac{11}{20}$	33(ii)	$\frac{11}{14}$	33(iii)	$\frac{11}{15}$	(34)	$\frac{2}{5}$
(35)	$\frac{23}{30}$	36(i)	$\frac{13}{30}$	36(ii)	$\frac{5}{6}$	36(iii)	$\frac{1}{60}$	(37)	$\frac{175}{931}$

(38)	$\frac{59}{130}$	(39)	$\frac{3}{5}$	40(i)	$\frac{19}{30}$	40(ii)	$\frac{11}{30}$	40(iii)	$\frac{1}{10}$
41(a)	$\frac{3}{4}$	41(b)	$\frac{1}{2}$	(42)	$\frac{5}{9}$	(43)	$\frac{13}{25}$	(44)	$\frac{7}{9}$
(45)	$\frac{1}{3}$	(46)	$\frac{29}{70}$	(47)	$\left(\frac{11}{16}\right)^4$	(48)	$\frac{1}{5}$	(49)(i)	$\frac{47}{120}$
(49)(ii)	$\frac{119}{120}$	(50)	(d) $\frac{7}{10}$	(51)	(b) $\frac{18}{95}$	(52)(i)	$\frac{2}{15}$	(52)(ii)	$\frac{3}{5}$
(52)(iii)	$\frac{4}{15}$	(52)(iv)	$\frac{11}{15}$						

Probability (Baye's Theorem)

ISC Class 12 Board Questions

With Answers

2000 to Sem 2-2022

1. A firm produces steel pipes in three plants A, B and C with daily production of 500, 1000 and 2000 units respectively. It is known that fractions of defective output produced by the three plants are respectively 0.005, 0.008 and 0.01. A pipe is selected at random from a day's total production and found to be defective. What is the probability that it came from the first plant?

   (ISC 2005, 2000)

2. An insurance company insured 6000 scooter drivers, 3000 car drivers and 9000 truck drivers. The probability of an accident involving a scooter, a car or a truck is 0.02, 0.06 and 0.3 respectively. One of the insured persons meets with an accident. Find the probability that he is a car driver. (ISC 2002)

3. A company has two plants which manufacture scooters. Plant **I** manufacture 80% of the scooters while plant **II** manufactures 20% of the scooters. At plant **I**, 85 out of 100 scooters are being of standard quality, while at plant **II** only 65 out of 100 scooters are rated as being of standard quality. If a scooter is of standard quality, what is the probability that it comes from plant **I**? (ISC 2006)

4. An insurance company insured 1500 scooter drivers, 2500 car drivers and 4500 truck drivers. The probability of a scooter, a car and a truck meeting with an accident is 0.01, 0.02 and 0.04 respectively. If one of the insured persons meets with an accident, find the probability that he is a scooter driver. (ISC 2007)

5. A class consists of 50 students out of which there are 10 girls. In the class 2 girls and 5 boys are rank holders in an examination. If a student is selected at random from the class and is found to be rank holder, what is the probability that the student selected is a girl?

   (ISC 2008)

6. An insurance company insured 4000 doctors, 8000 teachers and 12000 engineers. The probabilities of a doctor, a teacher and an engineer dying before the age of 58 years are 0.01, 0.03 and 0.05 respectively. If one of the insured persons dies before the age of 58 years, find the probability that he is a doctor. (ISC 2009)

7. A factory of three machines A, B and C producing 1500, 2500 and 3000 bulbs per day respectively. Machine A produces 1.5% defective bulbs, machine B produces 2% defective bulbs and machine C produces 2.5% defective bulbs. At the end of the day, a bulb is drawn at random and is found to be defective. What is the probability that this defective bulb has been produced by machine B? (ISC 2010)

8. Bag A contains 2 white, 1 black and and 3 red balls. Bag B contains 3 white, 2 black and 4 red balls and bag C contains 4 white, 3 black and 2 red balls. One bag is chosen at random, and 2 balls are drawn at random from that bag. If the randomly drawn balls happen to be red and black, what is the probability that both balls come from bag B? (ISC 2011)

9. In a class of 75 students, 15 are average, 45 are average and the rest below average achievers. The probability that an above average achieving student fails is 0.005, that an average achieving student fails is 0.05 and the probability of a below average achieving student failing is 0.15. If a student is known to have passed, what is the probability that he is a below average achiever? (ISC 2012)

10. For A, B, and C, the chances of being selected as the manager of a firm are 4: 1: 2, respectively. The probabilities for them to introduce a radical change in the marketing strategy are 0.3, 0.8 and 0.5 respectively. If a change takes place; find the probability that it is due to the appointment of B. (ISC 2013)

11. In a bolt factory, three machines A, B and C manufacture 25%, 35% and 40% of the total production respectively and of their respective outputs, 5%, 4% and 2% are defective. A bolt is drawn at random from the total production and it is found to be defective. Find the probability that it was manufactured by machine C. (ISC 2014)

12. Box I contains two white and three black balls and box II contains four white and one black balls and box III contains three white and four black balls. A dice having three red, two yellow and one green face, is thrown to select the box. If red face turns up, we pick up box I, if a yellow face turns up, we pick up box II, otherwise we pick up box III. Then we draw a ball from the selected box. If the ball drawn is white, what is the probability that the dice had turned up with a red face? (ISC 2015)

13. In an automobile factory, certain parts are to be fixed into the chassis in a section before it moves into another section. On a given day, one of the three persons A, B and C carries out this task. A has 45% chance, B has 35% chance and C has 20% chance of doing the task. The probability that A, B and C will take more than the allotted time is $\frac{1}{6}, \frac{1}{10}$ and $\frac{1}{20}$ respectively. If it is found that the time taken is more than the allotted time, what is the probability that A has done the task? (ISC 2016)

14. A fair die is rolled. If face one turns up, a ball is drawn from Bag A. If face 2 or 3 turns up, a ball is drawn from Bag B. If face 4 or 5 or 6 turns up, a ball is drawn from Bag C. Bag A contains 3 red and 2 white balls, Bag B contains 3 red, and 4 white balls and Bag C contains 4 red and 5 white balls. The die is rolled, a bag is picked up and a ball is drawn. If the drawn ball is red; what is the probability that it is drawn from Bag B? (ISC 2017)

15. Given three identical boxes A, B and C. Box A contains 2 gold and 1 silver coin, Box B contains 1 gold, and 2 silver coins and Box C contains 3 silver coins. A person chooses a box at random and takes out a coin. If the coin drawn is of silver, find the probability that it has been drawn from the Box which has the remaining two coins also of silver. (ISC 2019)

16. An insurance company insured 1000 scooter drivers, 2000 car drivers and 4000 truck drivers. The probability of accidents by scooter, car and truck drivers are 0.02, 0.05 and 0.03 respectively. If one of the insured persons meets with an accident, find the probability that he is a truck driver. (ISC Sem 2 - 2022)

Answer

1.	$\frac{5}{61}$	2.	0.06	3.	$\frac{68}{81}$	4.	$\frac{3}{49}$
5.	$\frac{2}{7}$	6.	$\frac{1}{22}$	7.	$\frac{20}{59}$	8.	$\frac{20}{53}$
9.	0.18	10.	$\frac{4}{15}$	11.	$\frac{16}{69}$	12.	$\frac{42}{113}$
13.	0.625	14.	$\frac{90}{293}$	15.	$\frac{1}{2}$	16.	$\frac{1}{2}$

Probability

Probability distribution and Binomial distribution

ISC Class 12 Board Questions

With Answers

1992 to 2020

1. In a large collection of bulbs, 3 out of 5 are daffodils and the rest are tulips. If they are planted at random, calculate the probability that in a row of five plants
   (i) all are daffodils.
   (ii) at least four are daffodils. (ISC 1992)

2. Mohan and Sohan play 12 games of chess. Mohan wins 6 games and Sohan 4 games. 2 games end in a draw. They agree to play three more games. Calculate the probability that out of these 3 games, 2 games end in a draw. (ISC 1993)

3. On an average, out of 12 games of chess played by A and B, A wins 6, B wins 4 and 2 games end in a tie. A and B play tournament of 3 games. Calculate the probability that
   (i) at least two games end in a tie.
   (ii) A and B win alternate games, no games end in a tie. (ISC 1994)

4. The probability density function y of a continuous variable x is given by $y = \frac{x}{k}, 0 \leq x \leq 2$ and $y = 0$ for all other values of x. Calculate the values of k and the probability that $x < 1$. (ISC 1994)

5. A pair of dice is thrown five times. If getting a doublet is considered a success, find the probability of 2 successes. (ISC 1995)

6. Assume that on an average, one telephone out of 10 is busy, six telephone numbers are randomly selected and called. Find the probability that four of them will be busy. (ISC 1997)

7. Assume that half the population are consumers of chocolate, so that the chance of an individual being a consumer is $\frac{1}{2}$. If 100 investigators interviewed 10 individuals each to see whether they are consumers, how many investigators would you expect to report that 3 individuals or less are consumers? (ISC 1998)

8. A and B play a game which A's chance of winning the game is $\frac{3}{5}$. In a series of 6 games, find the probability that A will win at least 4 games. (ISC 1999)

9. A die is tossed three times. Getting a '3' or a '5' is considered a success. Find the probability of at least two successes. (ISC 2004)

10. A coin is tossed five times. What is the probability of getting at least three heads? (ISC 2005)

11. Four dice are thrown simultaneously. If the occurrence of an odd number in a single die is considered a success, find the probability of at most 2 successes. (ISC 2006)

12. Assuming that on an average one telephone out of 10 is busy, seven telephone numbers are randomly selected and called. Find the probability that three of them will be busy. (ISC 2007)

13. In a binomial distribution, the sum of its mean and variance is 1.8. Find the probability of two successes if the event was conducted five times. (ISC 2007)

14. 8 coins are thrown simultaneously.
    (i) Show that the probability of getting at least 6 heads is $\frac{37}{256}$.
    (ii) What is the probability of getting at least three heads? (ISC 2008)

15. The mean and variance of a binomial distribution are 4 and 2 respectively. Find the probability of at least six successes. (ISC 2009)

16. Five bad eggs are mixed with 10 good ones. If three eggs are drawn one by one with replacement, find the probability distribution of the number of good eggs drawn. (ISC 2010)

17. A box contains 4 red and 5 black marbles. Find the probability distribution of the red marbles in a random draw of 3 marbles. Also, find the mean and standard deviation of the distribution. (ISC 2011)

18. The probability that a bulb produced by a factory will fuse in 100 days of use is 0.05. Find the probability that out of 5 such bulbs, after 100 days of use:
    (i) none fuses
    (ii) not more than one fuses.
    (iii) more than one fuses.
    (iv) at least one fuses. (ISC 2012)

19. If the sum and the product of the mean and variance of a binomial distribution are 1.8 and 0.8 respectively, find the probability distribution and the probability of at least one success. (ISC 2013)

20. On dialling certain telephone numbers, assume that on an average one telephone number out of 5 is busy, ten telephone numbers are randomly selected and dialled. Find the probability that at least three of them will be busy. (ISC 2014)

21. Five dice are thrown simultaneously. If the occurrence of an odd number in a single dice is considered a success, find the probability of maximum three successes. (ISC 2015)

22. The difference between mean and variance of a binomial distribution is 1 and the difference of their squares is 11. Find the distribution. (ISC 2016)

23. From a lot of six items containing two defective items, a sample of four items are drawn at random. Let the random variable X denote the number of defective items in the sample. If the sample is drawn without replacement, find:
    (a) the probability distribution of X
    (b) the mean of the distribution
    (c) the variance of the distribution (ISC 2018)

24. Determine the binomial distribution where mean is 9 and standard deviation is $\frac{3}{2}$. Also, find the probability of obtaining at most one success. (ISC 2019)

25. The probability that a bulb produced in a factory will fuse after 150 days of use is 0.05. Find the probability that out of five such balls bulbs:
    (i) none will fuse after 150 days of use
    (ii) not more than one will fuse after 150 days of use
    (iii) more than one will fuse after 150 days of use
    (iv) at least one will fuse after 150 days of use. (ISC 2020)

Answer

1(i)	$\frac{243}{3125}$	1(ii)	$\frac{1053}{3125}$	(2)	$\frac{5}{72}$	3(i)	$\frac{2}{27}$ (ii) $\frac{5}{36}$
(4)	$\frac{1}{4}$	(5)	$\frac{625}{3888}$	(6)	0.001215	(7)	17 only
(8)	$\frac{1701}{3125}$	(9)	$\frac{7}{27}$	(10)	$\frac{1}{2}$	(11)	$\frac{11}{16}$
(12)	0.023	(13)	$\frac{128}{625}$	14(i)	$\frac{37}{256}$ (ii) $\frac{219}{256}$	(15)	$\frac{37}{256}$

(16)	X	0	1	2	3		
	P(X)	$\frac{1}{27}$	$\frac{2}{9}$	$\frac{4}{9}$	$\frac{8}{27}$		
(17)	X	0	1	2	3		
	P(X)	$\frac{5}{42}$	$\frac{10}{21}$	$\frac{5}{14}$	$\frac{1}{21}$		

Mean = $1\frac{1}{3}$ Standard deviation = $\frac{\sqrt{5}}{3} = 0.745$

18(i)	0.77	(ii)	0.97	(iii)	0.03	(iv)	0.23

(19)	$\left(\frac{4}{5}+\frac{1}{5}\right)^5$, $\frac{2101}{3125}$	(20)	0.32	(21)	$\frac{13}{16}$	(22)	$\left(\frac{5}{6}+\frac{1}{6}\right)^{36}$

(23)	X	0	1	2
	P(X)	$\frac{1}{15}$	$\frac{8}{15}$	$\frac{6}{15}$
	mean	$\frac{4}{3}$	variance	$\frac{16}{45}$

(24)	$\left(\frac{3}{4}+\frac{1}{4}\right)^{12}$	$\frac{37}{4^{12}}$

(25)	i) $(0.95)^5$	ii) $(0.95)^4 \times 1.2$
	iii) $1 - (0.94)^4 \times 1.2$	iv) $1 - (0.95)^5$

Solutions of

Probability

(Addition and Multiplication Theorem, Conditional Probability)

ISC Class 12 Previous Years Board Questions

1993 to 2020

1. The probabilities a student getting I, II, III division in an examination are respectively $\frac{1}{10}, \frac{3}{5} and \frac{1}{4}$. Calculate the probability that the student fails in the examination. (ISC 1993)

Solution: There are four possibilities of the result of the examination of a student:

(1) He gets I division
(2) He gets II division
(3) He gets III division
(4) He fails in the examination

P (student gets I division) = P(A) $= \frac{1}{10}$

P (student get II division = P(B) $= \frac{3}{5}$

P (student get III division) $= \frac{1}{4}$

P (student fails in the examination) = P(D) $= x$ (let)

$\therefore$ P(A) + P(B) + P(C) + P(D) = 1

$\therefore \frac{1}{10} + \frac{3}{5} + \frac{1}{4} + x = 1$

$\therefore x = \frac{1}{20}$ Ans

2. Three shots are fired at a moving target. The probabilities of hitting the target at the first, second and third shots are $\frac{1}{2}, \frac{3}{10} and \frac{1}{10}$ respectively. Calculate the probability that the target gets hit. (ISC 1994)

Solution: The probability that first shot hits target $= \frac{1}{2}$

The probability that it does not hit at first shot $= 1 - \frac{1}{2} = \frac{1}{2}$

The probability of second shot to hit the target $= \frac{1}{2} \times \frac{3}{10} = \frac{3}{20}$, because it will hit in the

Second trial only when it does not hit at the first shot.

Similarly, the probability of third shot to hit the target $= \frac{1}{2} \times \frac{7}{10} \times \frac{1}{10} = \frac{7}{200}$

Hence total probability $= \frac{1}{2} + \frac{3}{20} + \frac{7}{200} = \frac{137}{200}$ Answer.

3. **A** speaks truth in 55% cases and B speaks truth in 75% cases. Determine the percentage of cases in which they are likely to contradict each other in stating the same fact. (ISC 1995)

Solution: Let A and B be the events that A and B speak truth respectively.

$\therefore P(A) = \frac{55}{100} = \frac{11}{20}, \; P(B) = \frac{75}{100} = \frac{3}{4}$

$\therefore P(A^c) =$ probability that A does not speak truth $= 1 - \frac{11}{20} = \frac{9}{20}$

$\therefore P(B^c) = 1 - \frac{3}{4} = \frac{1}{4}$

They will contradict each other if either (i) A speaks truth and B does not speak truth or

Or (ii) B speaks truth and A does not speaks truth.

By the theorem of total probability, the probability that A and B contradict each other

$P = P(A)\, P\,(B^c) + P\,(A^c) P\,(B) = \frac{11}{20} \times \frac{1}{4} + \frac{3}{4} \times \frac{9}{20} = \frac{19}{40}$

Hence the required percentage of cases in which they are likely to contradict each other

$= \frac{19}{40} \times 100 = 47.5\%$ Answer.

4. Find the probability of drawing an ace or spade or both from a deck of cards. (ISC 1996)

Solution: Let A be the event of drawing an ace, B is the event of drawing a spade.

Events are not mutually exclusive as one ace is a spade also.

P(A) = probability of drawing an ace = $\frac{4}{52}$

P(B) = probability of drawing a spade = $\frac{13}{52}$

P (A ∩ B) = probability of drawing an ace which is a spade also = $\frac{1}{52}$

As P (A∪ B) = P(A) + P(B) – P(A∩ B)

P (A ∪ B) = $\frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$ Answer.

5. A problem in mathematics is given to three students **A**, **B** and **C**. The chances of solving it by **A**, **B** and **C** are respectively $\frac{1}{3}, \frac{1}{4}\ and\ \frac{1}{5}$ .Calculate the probability that the problem will be solved.

(ISC 1997, 2003)

Solution: Let A be the event that student A will solve the problem.

Similarly, B and C be the event that the student B and C solve the problem respectively.

Then, P(A) $= \frac{1}{3}$, P(B) $= \frac{1}{4}$, P(C) $= \frac{1}{4}$

∴ P (A^c) = Probability that A will not be able to solve the problem

$$= 1 - P(A) = 1 - \frac{1}{3} = \frac{2}{3}$$

Similarly, $P(B^c) = 1 - \frac{1}{4} = \frac{3}{4}$ and $P(C^c) = 1 - \frac{1}{5} = \frac{4}{5}$

The probability that none A, B, C can solve the problem

$$= P(A^c).P(B^c).P(C^c) = \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} = \frac{2}{5}$$

Hence the required probability that the problem will be solved

$$= 1 - \frac{2}{5} = \frac{3}{5}$$ Answer.

6. In a given race, the odds are in favour of four horses **A**, **B**, **C**, **D** are 1:3, 1:4, 1:5, 1:6 respectively. Assuming that a dead heat is impossible, find the chance that one of them wins the race.

(ISC 1998)

Solution: let A, B, C and D be the event that the A, B, C, D win the race respectively.

$$\therefore P(A) = \frac{1}{4},\ P(B) = \frac{1}{5},\ P(C) = \frac{1}{6},\ P(D) = \frac{1}{7}$$

$$\therefore\ P(A^c) = 1 - \frac{1}{4} = \frac{3}{4},\ P(B^c) = \frac{4}{5},\ P(C^c) = \frac{5}{6},\ P(D^c) = \frac{6}{7}$$

The chance that one of them wins the race

$$= P(A \cap B^c \cap C^c \cap D^c) + P(A^c + B \cap C^c \cap D^c) + P(A^c \cap B^c \cap C \cap D^c) + P(A^c \cap B^c \cap C^c \cap D)$$

$$= P(A).P(B^c).P(C^c).P(D^c) + P(A^c).P(B).P(C^c).P(D^c) + P(A^c).P(B^c).P(C).P(D^c) + P(A^c).P(B^c).P(C^c).P(D)$$

$$= \frac{1}{4} \times \frac{4}{5} \times \frac{5}{6} \times \frac{6}{7} + \frac{3}{4} \times \frac{1}{5} \times \frac{5}{6} \times \frac{6}{7} + \frac{3}{4} \times \frac{4}{5} \times \frac{1}{6} \times \frac{6}{7} + \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} \times \frac{1}{7} = \frac{1}{7} + \frac{3}{28} + \frac{3}{35} + \frac{1}{14}$$

$$= \frac{57}{140}$$ Answer

7. The probability that a boy will not pass MBA examination is $\frac{3}{5}$ and that a girl will not pass is $\frac{4}{5}$. Calculate the probability that at least one of them passes the examination. (ISC 1999)

Solution: Let B be the event that the boy passes the MBA exam.

and G be the event that the girl passes the exam.

So, the probability that the boy does not pass the MBA exam = $P(B^c) = \frac{3}{5}$ and $P(G^c) = \frac{4}{5}$

$P(B) = 1 - \frac{3}{5} = \frac{2}{5}$ and $P(G) = 1 - \frac{4}{5} = \frac{1}{5}$

So, the probability that at least one of them passes the examination

$= P(B).P(G) + P(B^c).P(G) + P(B).P(G^c) = \frac{2}{5} \times \frac{1}{5} + \frac{3}{5} \times \frac{1}{5} + \frac{2}{5} \times \frac{4}{5} = \frac{13}{25}$ Answer

8. A card is drawn at random from a pack of 52 playing cards. What is the probability that the card drawn is neither a spade or a queen? (ISC 2000, 2006)

Solution: This is a case of non-mutually exclusive event.

The probability of card is spade = $P(A) = \frac{13}{52}$

The probability that the card is of queen = $P(B) = \frac{4}{52}$ and $P(A \cap B) = \frac{1}{52}$

The probability that the card is either spade or queen

$= P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$

∴ the probability that the card drawn is neither a spade nor a queen

$= 1 - \frac{4}{13} = \frac{9}{13}$ Answer

9. A bag contains 20 balls marked 1 to 20. One ball is drawn at random from the bag. What is the probability that the ball drawn is marked with a number which is a multiple of 5 or 7. (ISC 2001)

Solution: Bag contains 20 balls marked 1 to 20.

Multiples of 5 from 1 to 20 are 5, 10, 15,20 and multiples of 7 from 1 to 20 are 7, 14

So favourable outcomes = {5, 7, 10, 14, 15, 20}

No of favourable outcomes = 6, No of all possible outcomes = 20

∴ the probability that the ball drawn is marked with a multiple of 5 or 7 = $\frac{6}{20} = \frac{3}{10}$ Ans.

10. A problem in mathematics is given to four students A, B, C and D. Their chances of solving the problems are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}\ and\ \frac{1}{5}$ respectively. What is the probability that the problem will be solved? (ISC 2001)

Solution: Let A, B, C, D be the events that A, B, C, D can respectively solve the problem.

$\therefore P(A) = \frac{1}{2},\ P(B) = \frac{1}{3},\ P(C) = \frac{1}{4},\ P(D) = \frac{1}{5}$

$\therefore P(A^c)$ = probability that student A can not solve the problem $= 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}$

Similarly, $P(B^c) = 1 - \frac{1}{3} = \frac{2}{3}, P(C^c) = 1 - \frac{1}{4} = \frac{3}{4}, P(D^c) = 1 - \frac{1}{5} = \frac{4}{5}$

Probability that none A, B, C, D can solve the problem $= P(A^c).P(B^c).P(C^c).P(D^c)$

$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} = \frac{1}{5}$

Hence, the required probability that the problem will be solved $= 1 - \frac{1}{5} = \frac{4}{5}$ Answer

11. A pair of dice is thrown. If two numbers appearing on them is different, find the probability that the sum of the numbers appearing is 6. (ISC 2001)

Solution: Favourable outcomes = {(1,5), (2,4), (4, 2), (5,1)} So no. Of favourable outcomes = 4

No. of all possible outcomes = 36

$\therefore$ probability that the sum of numbers appearing on a pair of dice is 6

$= \frac{No.of\ \ favourable\ outcomes}{No.\ of\ total\ outcomes} = \frac{4}{36} = \frac{1}{9}$ Answer

12. There are 10 persons who are to be seated together around a circular table. Find the probability that the two particular persons will always seat together. (ISC 2002)

Solution: If 'n' objects are arranged in a circle, the total no. of ways in which they can be arranged $= (n-1)!$ Here $n = 10$. So, total no. of ways, in which 10 persons can be seated around a circular table $= (10-1)! = 9!$

If two particular persons always sit together, then we have to arrange only 9 persons around a circular table.

$\therefore$ no. of ways in which 9 persons can be seated around a circular table $= (9-1)! = 8!$

But these two particular persons can interchange their seats. So no. of favourable cases = $2 \times 8!$

Probability that two particular persons always sit together out of 10 persons around a circular table

$= \frac{no.of\ fabourable\ cases}{no.of\ total\ cases} = \frac{2 \times 8!}{9!} = \frac{2 \times 8!}{9 \times 8!} = \frac{2}{9}$ Answer

13. One number is chosen at random from the numbers 1 to 21. Find the probability that it may be a prime number. (ISC 2003)

Solution: prime numbers between 1 to 21 are 2, 3, 5, 7, 11, 13, 17, 19

Number of favourable outcomes = 8, number of all possible outcomes = 21

$\therefore$ the probability that the number is a prime number $= \frac{8}{21}$ Answer

14. A bag has 4 red and 5 black, a second bag has 3 red and 7 black balls. One ball is drawn from 1st bag and two balls from 2nd bag. Find the probability that two balls are black, and one is red. (ISC 2004)

Solution:

B_1
4 red
5 black

B_2
3 red
7 black

(i) Either we can draw 1 black ball from B_1 and 1 red and 1 black from B_2.

$$P_1 = \frac{^5C_1}{^9C_1} \times \frac{^3C_1 \times ^7C_1}{^{10}C_2} = \frac{5}{9} \times \frac{3 \times 7}{45} = \frac{7}{27}$$

(ii) Either we can draw 1 red ball from B_1 and 2 black balls from B_2.

$$P_2 = \frac{^4C_1}{^9C_1} \times \frac{^7C_2}{^{10}C_2} = \frac{4}{9} \times \frac{21}{45} = \frac{28}{135}$$

Event (i) and event (ii) are mutually exclusive events i,e., happening of both events simultaneously is impossible. One of the two events will occur at one time.

Probability of event of drawing 2 black balls and one red ball from the bags

$$= P_1 + P_2 = \frac{7}{27} + \frac{28}{135} = \frac{63}{135} = \frac{7}{15}$$ Answer

15. A candidate is selected for interview of management trainees in 3 companies. For the first company, there are 12 candidates, for the second there are 15 candidates and for the third, there are 10 candidates. Find the probability that he is selected by at least one of the companies. (ISC 2005)

Solution: Let A, B, C be the events that the candidate is selected from the first, second and third company respectively.

$\therefore$ the probability that the candidate is selected from the first company $= P(A) = \frac{1}{12}$

Similarly, $P(B) = \frac{1}{15}$, $P(C) = \frac{1}{10}$

$\therefore$ the probability that the candidate is not selected from the first company $= P(\bar{A}) = 1 - \frac{1}{12} = \frac{11}{12}$

and $P(\bar{B}) = \frac{14}{15}, P(\bar{C}) = \frac{9}{10}$

$\therefore$ P (he is selected in at least one of the companies) $= 1 - P(\bar{A})P(\bar{B})P(\bar{C}) = 1 - \frac{11}{12}.\frac{14}{15}.\frac{9}{10} = \frac{23}{110}$ Ans.

16. The probability of A, B and C solving a problem are $\frac{1}{3}, \frac{2}{7}, \frac{3}{8}$ respectively. If all try and solve the problem simultaneously, find the probability that only one of them solve it. (ISC 2006)

Solution: Let A be the event that A can solve the problem. Similarly, events B and C for B and C to solve the problem. Then $P(A) = \frac{1}{3},\ P(B) = \frac{2}{7},\ P(C) = \frac{3}{8}$

$\therefore P(A^c) =$ the probability that A can not solve the problem $= 1 - \frac{1}{3} = \frac{2}{3}$

Similarly, $P(B^c) = 1 - \frac{2}{7} = \frac{5}{7},\ P(C^c) = 1 - \frac{3}{8} = \frac{5}{8}$

Exactly one of them can solve the problem if either only A or only B only C can solve it.

Hence required probability that exactly one of A, B, C will solve the problem

$= P(A \cap B^c \cap C^c) + P(A^c \cap B \cap C^c) + P(A^c \cap B^c \cap C)$

$= P(A).P(B^c).P(C^c) + P(A^c).P(B).P(C^c) + P(A^c).P(B^c).P(C)$

$= \frac{1}{3} \times \frac{5}{7} \times \frac{5}{8} + \frac{2}{3} \times \frac{2}{7} \times \frac{5}{8} + \frac{2}{3} \times \frac{5}{7} \times \frac{3}{8} = \frac{25}{168} + \frac{5}{42} + \frac{5}{28} = \frac{25}{56}$ Answer.

17. **A** and **B** throw two dice each. If **A** gets a sum of 9 on his two dice, then find the probability of **B** getting a higher sum. (ISC 2006)

Solution: B may throw numbers whose sum are 10, 11, 12.

Favourable outcomes = {(4,6), (5, 5), (6, 4), (5, 6), (6, 5), (6, 6)}

$\therefore$ number of favourable outcomes = 6

Number of all possible outcomes = 36

$\therefore$ the probability that the B getting higher sums $= \frac{6}{36} = \frac{1}{6}$ Answer.

18. Kamal and Monica appear for an interview for two vacancies. The probability of Kamal's selection is $\frac{1}{3}$ and that of Monica's selection is $\frac{1}{5}$. Find the probability that only one of them will be selected. (ISC 2007)

Solution: Let K be the event that the Kamal will be selected and

M is the event that the Monica is selected.

$\therefore P(K) = \frac{1}{3},\ P(M) = \frac{1}{5}$ and $P(K^c) = \frac{2}{3}, P(M^c) = \frac{4}{5}$

$\therefore$ the required probability $= P(K).P(M^c) + P(K^c).P(M) = \frac{1}{3} \times \frac{4}{5} + \frac{2}{3} \times \frac{1}{5} = \frac{2}{5}$ Answer.

19. The bag A contains 3 white and 2 black balls while the bag B contains 2 white and 5 black balls. One of the bags is chosen at random and a ball is drawn from it. What is the probability that the ball is white? (ISC 2007)

Solution:

Bag A	Bag B
3 white 2 black	2 white 5 black

White ball can be drawn either from bag A or from the bag B.

Let A be the event that the white ball is drawn from the bag A and B be the event that the white ball is drawn from the bag B.

(i) Probability of selecting bag A $= \frac{1}{2}$

$\therefore$ probability of selecting bag A and the drawing one white ball from bag A $= \frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$

(ii) Probability of selecting bag B $= \frac{1}{2}$

$\therefore$ probability of selecting bag B and the drawing one white ball from bag B $= \frac{1}{2} \times \frac{2}{7} = \frac{1}{7}$

Hence, required probability of drawing a white ball $= \frac{3}{10} + \frac{1}{7} = \frac{31}{70}$ Answer

20. In a single throw of two dice find the probability of getting a total of at most 9. (ISC 2008)

Solution: Total no. of equally likely elementary events = 36

Two numbers whose sum is more than 9 may occur in the following outcomes:

{(5, 5), (6, 4), (4, 6), (5, 6), (6, 5), (6, 6)}

$\therefore$ the probability of getting sum more than 9 $= \frac{6}{36} = \frac{1}{6}$

So, required probability of getting a total of at most 9 $= 1 - \frac{1}{6} = \frac{5}{6}$ Answer

21. There are three urns A, B and C. Urn A contains 4 red balls and 3 black balls. Urn B contains 5 red balls and 4 black balls. Urn C consists of 4 red balls and 4 black balls. One ball is drawn from each of these urns. What is the probability that the three balls drawn consist of 2 red balls and 1 black ball? (ISC 2008)

Solution:

Urn A	Urn B	Urn C
4 red 3 black	5 red 4 black	4 red 4 black

Let R be the event of drawing a red ball and B be the event of drawing a black ball. Then three cases may arise as shown below.

(i) RRB $\Rightarrow P(RRB) = \frac{4}{7} \times \frac{5}{9} \times \frac{4}{8} = \frac{80}{7\times9\times8}$

(ii) RBR $\Rightarrow P(RBR) = \frac{4}{7} \times \frac{4}{9} \times \frac{4}{8} = \frac{64}{7\times9\times8}$

(iii) BRR $\Rightarrow P(BRR) = \frac{3}{7} \times \frac{5}{9} \times \frac{4}{8} = \frac{60}{7\times9\times8}$

Hence, required probability that the three balls drawn consist of 2 red balls and 1 black ball

$= \frac{80}{7\times9\times8} + \frac{64}{7\times9\times8} + \frac{60}{7\times9\times8} = \frac{204}{7\times9\times8} = \frac{17}{42}$ Answer

22. Bag **A** contains 5 white 4 black balls, bag **B** contains7 white and 6 black balls. One ball is drawn from bag **A** and without noticing its colour, is put in the bag **B**. If the ball is then drawn from bag **B**, find the probability that it is black in colour. (ISC 2009)

Solution:

Bag A	Bag B
5 white 4 black	7 white 6 black

The ball transferred from bag A to bag B may be either white or black.

(i) A white ball is transferred from bag A to bag B.
Probability (A white ball is transferred from bag A to bag B and a black ball is drawn from bag B $= \frac{5}{9} \times \frac{6}{14} = \frac{5}{21}$

(ii) A black ball is transferred from bag A to bag B.
Probability (A black ball is transferred from bag A to bag B and a black ball is drawn from bag B $= \frac{4}{9} \times \frac{7}{14} = \frac{2}{9}$

Hence, required probability $= \frac{5}{21} + \frac{2}{9} = \frac{20}{63}$ Answer

23. An article manufactured by a company consists of two parts A and B. In this process of manufacture of part A, 9 out of 104 parts may be defective. Similarly, 5 out of 100 are likely to be defective in the manufacture of part B. Calculate the probability that the article manufactured will not be defective. (ISC 2009)

Solution: Probability (A to be defective) = $\frac{9}{104}$, Probability (A not to be defective) $= 1 - \frac{9}{104} = \frac{95}{104}$

Probability (B to be defective) = $= \frac{5}{100}$, Probability (B not to be defective) $1 - \frac{5}{100} = \frac{95}{100}$

$\therefore$ Probability that the article manufactured not to be defective $= \frac{95}{104} \times \frac{95}{100} = \frac{361}{416}$ Answer

24. Two horses are considered for a race. The probability of selection of first horse is $\frac{1}{4}$ and that of the second is $\frac{1}{3}$. What is the probability that :
(i) both of them will be selected.
(ii) only one of them will be selected.
(iii) none of them will be selected. (ISC 2009)

Solution: Let A and B respectively denote the event of selection of 1st horse and 2nd horse.

Given $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{3}$ $\therefore P(A^c) = 1 - \frac{1}{4} = \frac{3}{4}$, $P(B^c) = 1 - \frac{1}{3} = \frac{2}{3}$

(i) Probability (both of them will be selected) $P(A).P(B) = \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$

(ii) Probability (only one of them will be selected $= P(A \cap B^c) + P(A^c \cap B)$
$= P(A).P(B^c) + P(A^c).P(B) = \frac{1}{4} \times \frac{2}{3} + \frac{3}{4} \times \frac{1}{3} = \frac{5}{12}$

(iii) Probability (none of them will be selected) $= P(A^c \cap B^c) = P(A^c).P(B^c) = \frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$ Ans.

25. A fair die is thrown once. What is the probability that either an even number or a number greater than three will turn up? (ISC 2010)

Solution: Sample space $= \{1, 2, 3, 4, 5, 6\}$

Let A be the event of getting even number and B be the event of getting a number greater than 3. $\therefore$ A = {2, 4, 6}, B = {4, 5, 6} $\therefore$ P(A) = $\frac{3}{6} = \frac{1}{2}$ and P(B) = $\frac{3}{6} = \frac{1}{2}$ and P (A $\cap$ B) = $\frac{2}{6} = \frac{1}{3}$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{2} + \frac{1}{2} - \frac{1}{3} = 1 - \frac{1}{3} = \frac{2}{3}$$ Answer.

26. Akhil and Ajay appear for an interview for two vacancies. The probability of Akhil's selection is $\frac{1}{4}$ and Ajay's selection is $\frac{2}{3}$.Find the probability that the only one of them will be selected. (ISC 2010)

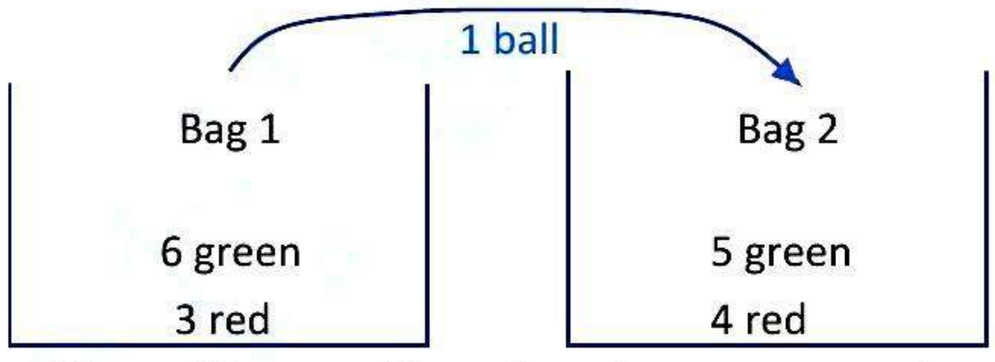
Solution: Let A be the event that the Akhil will be selected, and B be the event that the Ajay will be selected. $\therefore P(A) = \frac{1}{4},\ P(B) = \frac{2}{3}$ $\therefore P(A^c) = 1 - \frac{1}{4} = \frac{3}{4},\ P(B^c) = 1 - \frac{2}{3} = \frac{1}{3}$

$\therefore$ required probability that only one of them will be selected

$$= P(A \cap B^c) + P(A^c \cap B) = P(A).P(B^c) + P(A^c).P(B) = \frac{1}{4} \times \frac{1}{3} + \frac{3}{4} \times \frac{2}{3} = \frac{1}{12} + \frac{6}{12} = \frac{7}{12}$$ Ans.

27. There are two bags. One bag contains six green and three red balls. The second bag contains five green and four red balls. One ball is transferred from the first bag to the second bag. Then one ball is drawn from the second bag. Find the probability that it is a red ball. (ISC 2010)

Solution:



The ball transferred from 1st bag to 2nd, can be either green or red.

Case 1. Green ball is transferred from the 1st bag to the 2nd.

Probability of transferring one green ball from the 1st to the 2nd = $\frac{6}{9} = \frac{2}{3}$

After transferring, no. of green balls in 2nd = 6, and total no. of balls in the 2nd bag = 10

Probability of drawing a red ball from 2nd bag = $\frac{4}{10} = \frac{2}{5}$

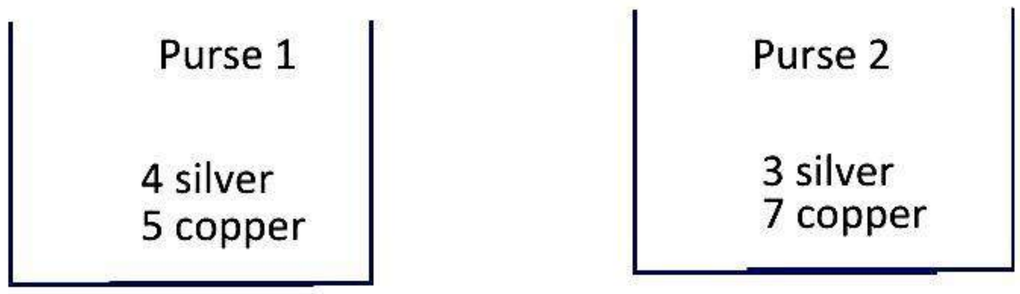
$\therefore$ Probability of transferring one green ball from 1st bag to 2nd and then drawing a red ball from the 2nd bag = $\frac{2}{3} \times \frac{2}{5} = \frac{4}{15}$

Case 2. Similarly, probability of transferring one red ball from 1st bag to 2nd bag and then drawing a red ball from the 2nd bag = $\frac{3}{9} \times \frac{5}{10} = \frac{1}{6}$

$\therefore$ the required probability $= \frac{4}{15} + \frac{1}{6} = \frac{8+5}{30} = \frac{13}{30}$ Answer.

28. A purse contains 4 silver and 5 copper coins. A second purse contains 3 silver and 7 copper coins. If a coin is taken out at random from one of the purses, what is the probability that it is a copper coin? (ISC 2011)

Solution:



Copper coin can be drawn either from 1st purse or from the 2nd purse.

(i) Probability of selecting the first purse = $\frac{1}{2}$

Probability of getting a copper coin from the 1st purse = $\frac{5}{9}$

∴ probability of selecting the first purse and then getting one copper coin from it

$$= \frac{1}{2} \times \frac{5}{9} = \frac{5}{18}$$

(ii) Probability of selecting the second purse = $\frac{1}{2}$

∴ probability of getting a copper coin from the 2nd purse = $\frac{7}{10}$

∴ probability of selecting the 2nd purse and then getting one copper coin from it

$$= \frac{1}{2} \times \frac{7}{10} = \frac{7}{20}$$

Hence the required probability of getting a copper coin = $\frac{5}{18} + \frac{7}{20} = \frac{50+63}{180} = \frac{113}{180}$ Ans.

29. Aman and Bhuvan throw a pair of dice alternately. In order to win, they have to get a sum of 8. Find their respective probabilities of winning if Aman starts the game. (ISC 2011)

Solution: Total no. of possible cases, when a pair of dice is thrown = 36

Favourable outcomes (sum of 8) = {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)}

∴ probability of getting sum of 8 = $\frac{5}{36}$ and probability of not getting sum of 8 = $1 - \frac{5}{36} = \frac{31}{36}$

1st case: When Aman wins the game

$$P(A) = \left(\frac{5}{36}\right) + \left(\frac{31}{36} \times \frac{31}{36} \times \frac{5}{36}\right) + \left(\frac{31}{36} \times \frac{31}{36} \times \frac{31}{36} \times \frac{31}{36} \times \frac{5}{36}\right) + \ldots\ldots\ldots$$

$$= \frac{5}{36}\left[1 + \left(\frac{31}{36}\right)^2 + \left(\frac{31}{36}\right)^4 + \cdots\right]$$

$$= \frac{5}{36} \cdot \frac{1}{1-\left(\frac{31}{36}\right)^2} = \frac{5}{36} \cdot \frac{1}{1-\frac{961}{1296}} = \frac{5}{36} \cdot \frac{1296}{335} = \frac{36}{67}$$ Answer.

2nd case: When Bhuban wins the game

$$P(B) = 1 - P(A) = 1 - \frac{36}{67} = \frac{31}{67}$$ Answer.

30. A bag contains 8 red and 5 white balls. Two successive draws of 3 bulbs are made at random from the bag without replacements. Find the probability that the first draw yields 3 white balls and the second draw 3 red balls. (ISC 2012)

Solution: Let W be the event that the 1st draw yields 3 white balls and R be the event that the 2nd draw yields 3 red balls.

1st draw: $P(W) = \frac{^5C_3}{^{13}C_3} = \frac{10}{286} = \frac{5}{143}$

2nd draw: $P(R) = \frac{^8C_3}{^{10}C_3} = \frac{56}{120} = \frac{7}{15}$

Bag
8 red
5 White

The required total probability $= P(W).P(R|W) = \frac{5}{143} \times \frac{7}{15} = \frac{7}{429}$ Answer.

31. Bag A contains three red and four white balls; Bag B contains two red and three white balls. If one ball is drawn from bag A and two balls from bag B, find the probability that:

(i) one ball is red, and two balls are white.

(ii) all the three balls are of same colour. (ISC 2013)

Solution:

Bag A → 1 ball
3 Red
4 White

Bag B → 2 balls
2 Red
3 White

(i) Probability (1 red ball from bag A and 2 white balls from bag B) $= \frac{^3C_1}{^7C_1}.\frac{^3C_2}{^5C_2} = \frac{9}{70}$

Probability (1 white ball from bag A and 1 red and 1 white ball from bag B) $= \frac{^4C_1}{^7C_1}.\frac{^2C_1 \cdot {}^3C_1}{^5C_2} = \frac{24}{70}$

Required probability (1 red ball and 2 white balls) $\frac{9}{10} + \frac{24}{70} = \frac{33}{70}$

(ii) Probability (all of same colour) = P (all three red balls) + P (all three white balls)

$= \frac{^3C_1}{^7C_1}.\frac{^2C_2}{^5C_2} + \frac{^4C_1}{^7C_1}.\frac{^3C_2}{^5C_2} = \frac{3}{7}.\frac{1}{10} + \frac{4}{7}.\frac{3}{10} = \frac{15}{70} = \frac{3}{14}$ Answer.

32. Three persons Aman, Bipin and Mohan attempt a mathematics problem independently. The odds in favour of Aman and Mohon solving the problem are 3 : 2 and 4 : 1 respectively and the odds against Bipin solving the problem are 2 : 1. Find:

(i) the probability that all the three will solve the problem.

(ii) the probability that problem will be solved. (ISC 2013)

Solution: Let A, B and C be the events of solving the problem by Aman, Bipin and Mohan respectively.

$\therefore\ P(A) = \frac{3}{5},\ P(B) = \frac{1}{3},\ P(C) = \frac{4}{5}\quad and\quad P(A^c) = \frac{2}{5}, P(B^c) = \frac{2}{3}, P(C^c) = \frac{1}{5}$

(i) Probability (all the three will solve the problem) = $P(A \cap B \cap C) = P(A).P(B).P(C)$

$= \frac{3}{5}.\frac{1}{3}.\frac{4}{5} = \frac{4}{25}$ Answer

(ii) Probability that the problem will be solved $= 1 - P(A^c \cap B^c \cap C^c)$

$= 1 - P(A^c).P(B^c).P(C^c) = 1 - \frac{2}{5} \times \frac{2}{3} \times \frac{1}{5} = \frac{71}{75}$ Answer

33. In a college 70% students pass in Physics, 75% pass in mathematics and 10% students fail in both. One student is chosen at random. What is the probability that:

(i) he passes in Physics and Mathematics.

(ii) he passes in Mathematics given that he passes in Physics.

(iii) he passes in Physics given that he passes in Mathematics. (ISC 2014)

Solution: Let A and B be the events that the student passed in Physics and Mathematics respectively.

$$\therefore \ P(A) = \frac{70}{100} = \frac{7}{10} \ , \ P(B) = \frac{75}{100} = \frac{3}{4} \ , \ P(A^c \cap B^c) = \frac{10}{100} = \frac{1}{10}$$

$$\therefore 1 - P(A \cup B) = \frac{1}{10} \quad \Rightarrow P(A \cup B) = \frac{9}{10}$$

(i) Probability (he passes in Physics and Mathematics)

$$= P(A \cap B) = P(A) + P(B) - P(A \cup B) = \frac{7}{10} + \frac{3}{4} - \frac{1}{10} = \frac{11}{20}$$ Answer

(ii) Probability (he passes in Mathematics given that he passes in Physics)

$$= P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{\frac{11}{20}}{\frac{7}{10}} = \frac{11}{14}$$ Answer

(iii) Probability (he passes in Physics given that he passes in Mathematics)

$$= P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{11}{20}}{\frac{3}{4}} = \frac{11}{15}$$ Answer

34. A bag contains 5 white and 4 black balls, and another bag contains 7 white and 9 black balls. A ball is drawn from the first bag and two balls are drawn from the second bag. What is the probability of drawing one white and two black balls? (ISC 2014)

Solution: Required probability

= P (1 white from 1st bag and two black from 2nd bag) + P(1 black from 1st bag and 1 white and 1 black from 2nd bag) = $\frac{{}^5C_1}{{}^9C_1} \cdot \frac{{}^9C_2}{{}^{16}C_2} + \frac{{}^4C_1}{{}^9C_1} \cdot \frac{{}^7C_1 \cdot {}^9C_1}{{}^{16}C_2} = \frac{5}{9} \cdot \frac{3}{10} + \frac{4}{9} \cdot \frac{7 \times 9}{120} = \frac{1}{6} + \frac{7}{30} = \frac{2}{5}$ Answer

35. An urn contains 2 white and 2 black balls. A ball is drawn at random. If it is white, it is not replaced into the urn. Otherwise, it is replaced with another ball of the same colour. The process is repeated. Find the probability that the third ball drawn is black. (ISC 2015)

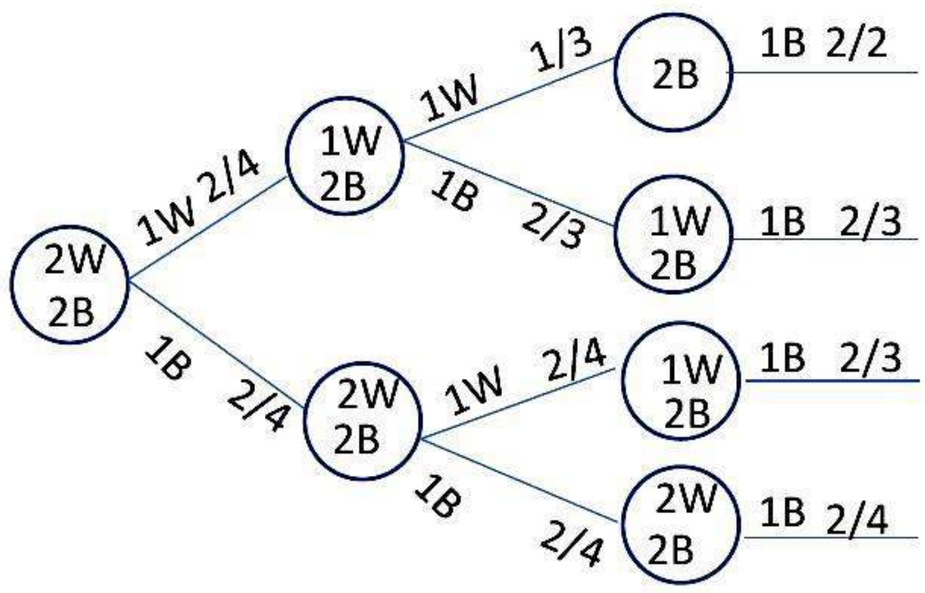
Solution: Let W be the event of drawing a white ball and B be the event of drawing a black ball. Then four cases may arise:

Case 1: WWB ⇒ P(WWB) = $\frac{2}{4} \times \frac{1}{3} \times \frac{2}{2} = \frac{1}{6}$

Case 2: WBB ⇒ $P(WBB) = \frac{2}{4} \times \frac{2}{3} \times \frac{2}{3} = \frac{2}{9}$

Case 3: BWB ⇒ $P(BWB) = \frac{2}{4} \times \frac{2}{4} \times \frac{2}{3} = \frac{1}{6}$

Case 3: BBB ⇒ $P(BBB) = \frac{2}{4} \times \frac{2}{4} \times \frac{2}{4} = \frac{1}{8}$



Hence, required probability = P(WWB) + P(WBB) + P(BWB) + P(BBB) = $\frac{1}{6} + \frac{2}{9} + \frac{1}{6} + \frac{1}{8} = \frac{49}{72}$ Ans.

36. Three persons A, B and C shoot to hit a target. If A hits the target four times in five trials, B hits it three times in four trials and C hits it in two times in three trials, find the probability that:

(i) exactly two persons hit the target.

(ii) at least two persons hit the target.

(iii) none hit the target. (ISC 2015)

Solution: Let A, B, C be the events that the A, B, C hit the target respectively.

$$\therefore P(A) = \frac{4}{5},\ P(B) = \frac{3}{4},\ P(C) = \frac{2}{3} \quad \therefore\ P(A') = 1 - \frac{4}{5} = \frac{1}{5},\ P(B') = \frac{1}{4},\ P(C') = \frac{1}{3}$$

(i) P (exactly two persons hit the target)

$= P(A \cap B \cap C') + P(A \cap B' \cap C) + P(A' \cap B \cap C)$

$= P(A).P(B).P(C') + P(A).P(B').P(C) + P(A').P(B).P(C)$

$= \frac{4}{5} \times \frac{3}{4} \times \frac{1}{3} + \frac{4}{5} \times \frac{1}{4} \times \frac{2}{3} + \frac{1}{5} \times \frac{3}{4} \times \frac{2}{3} = \frac{13}{30}$ Answer

(ii) P (at least two persons hit the target)

= P (exactly two persons hit the target) + P (all three persons hit the target)

$= \frac{13}{30} + P(A).P(B).P(C) = \frac{13}{30} + \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3} = \frac{25}{30} = \frac{5}{6}$ Answer

(iii) P (none hit the target) = P(A'). P(B'). P(C') = $\frac{1}{5} \times \frac{1}{4} \times \frac{1}{3} = \frac{1}{60}$ Answer

37. A committee of 4 persons has to be chosen from 8 boys and 6 girls, consisting of at least one girl. Find the probability that the committee consists of more girls than boys. (ISC 2016)

Solution: There must be at least one girl in the committee.

So, no of all possible ways that the committee can be done

$$= {}^{8}C_3 \times {}^{6}C_1 + {}^{8}C_2 \times {}^{6}C_2 + {}^{8}C_1 \times {}^{6}C_3 + {}^{8}C_0 \times {}^{6}C_4 = 336 + 420 + 160 + 15 = 931$$

Let A be the event of getting more girls than boys.

$$n(A) = {}^{8}C_1 \times {}^{6}C_3 + {}^{8}C_0 \times {}^{6}C_4 = 160 + 15 = 175$$

$\therefore$ required probability = P(A) = $\frac{175}{931} = 0.188$ Answer

38. An urn contains 10 white and 3 black balls while another urn contains 3 white and 5 black balls. Two balls are drawn from the first urn and put into the second urn and then a ball is drawn from the second urn. Find the probability that the ball drawn from the second urn is a white ball. (ISC 2016)

Solution: Three cases arise for transferring two balls from 1st to the 2nd urn and then drawing one ball from the 2nd urn. (Case 1) two white balls, (case 2) 1 white and 1 black ball, (case 3) 2 black balls.

Required probability = P (case 1) + P (case 2) + P (case 3) $= \frac{{}^{10}C_2}{{}^{13}C_2} \times \frac{{}^{5}C_1}{{}^{10}C_1} + \frac{{}^{10}C_1 \times {}^{3}C_1}{{}^{13}C_2} \times \frac{{}^{4}C_1}{{}^{10}C_1} + \frac{{}^{3}C_2}{{}^{13}C_2} \times \frac{{}^{3}C_1}{{}^{10}C_1}$

$= \frac{45}{78} \times \frac{5}{10} + \frac{10 \times 3}{78} \times \frac{4}{10} + \frac{3}{78} \times \frac{3}{10} = \frac{15}{52} + \frac{2}{13} + \frac{3}{260} = \frac{59}{130}$ Answer

39. A problem is given to three students whose chances of solving it are $\frac{1}{4}, \frac{1}{5}$, and $\frac{1}{3}$ respectively. Find the probability that the problem is solved. (ISC 2017)

Solution: Let $P(A) = \frac{1}{4}, P(B) = \frac{1}{5}, P(C) = \frac{1}{3}$. $\therefore P(A') = 1 - \frac{1}{4} = \frac{3}{4}$, $P(B') = \frac{4}{5}, P(C') = \frac{2}{3}$.

$\therefore$ Probability (the problem is solved) $= 1 - P(A').P(B').P(C') = 1 - \frac{3}{4} \times \frac{4}{5} \times \frac{2}{3} = \frac{3}{5}$ Answer

40. In a class of 60 students, 30 opted for mathematics, 32 opted for Biology and 24 opted for both mathematics and Biology. If one of these students is selected at random, find the probability that:
    (i) the student opted for mathematics or Biology.
    (ii) the student has opted neither mathematics nor biology.
    (iii) the student has opted Mathematics but not biology. (ISC 2017)

Solution: Total no of students in the class = 60

Let M denote the students opted Mathematics and B denote the students opted Biology.

$\therefore$ n(M) = 30, n(B) = 32 and n (M $\cap$ B) = 24

$\therefore P(M) = \frac{30}{60}$, $P(B) = \frac{32}{60}$, $P(M \cap B) = \frac{24}{60}$

(i) P (student opted mathematics or Biology) $= P(M \cup B) = P(M) + P(B) - P(M \cap B)$
$= \frac{30}{60} + \frac{32}{60} - \frac{24}{60} = \frac{38}{60} = \frac{19}{30}$ Answer

(ii) P (student opted neither mathematics nor biology) $= P(M' \cap B') = 1 - P(M \cup B)$
$= 1 - \frac{19}{30} = \frac{11}{30}$ Answer

(iii) P (student opted Mathematics but not Biology) $= P(M \cap B') = P(M) - P(M \cap B)$
$= \frac{30}{60} - \frac{24}{60} = \frac{6}{60} = \frac{1}{10}$ Answer

41. If A and B are events such that P(A) = $\frac{1}{2}$, P(B) = $\frac{1}{3}$ and P(A$\cap$B) = $\frac{1}{4}$, then find:
    (a) P(A/B)
    (b) P(B/A) (ISC 2018)

Solution: (a) $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{1}{3}} = \frac{3}{4}$ Answer (b) $P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{2}{4} = \frac{1}{2}$ Answer

42. In a race, the probabilities of A and B winning the race are $\frac{1}{3}$ and $\frac{1}{6}$ respectively. Find the probability of neither of them winning the race. (ISC 2018)

Solution: Probability (A wins the race) = P(A) = $\frac{1}{3}$, Probability (B wins the race) = P(B) = $\frac{1}{6}$

$\therefore$ $P(A') = 1 - \frac{1}{3} = \frac{2}{3}$, and $P(B') = 1 - \frac{1}{6} = \frac{5}{6}$

$\therefore$ P (neither of them winning the race) = P(A'). P(B') = $\frac{2}{3} \times \frac{5}{6} = \frac{5}{9}$ Answer

43. A speaks the truth in 60% of the cases, while B in 40% of the cases. In what percent of cases are they likely to contradict each other in stating the same fact? (ISC 2018)

Solution: Let A be the event that A speaks truth and B be the event that the B speaks truth.

$$\therefore P(A) = \frac{60}{100} = \frac{3}{5}, \quad P(B) = \frac{40}{100} = \frac{2}{5}$$

$$\therefore \quad P(A^c) = \text{ probability that A does not speak truth} = 1 - \frac{3}{5} = \frac{2}{5} \text{ and } P(B^c) = \frac{3}{5}$$

They contradict each other if either (i) A speaks truth and B does not speak truth or (ii) A does not speak truth and B speaks truth.

So, required probability (A and B contradict each other) $= P(A).P(B^c) + P(A^c).P(B)$

$$= \frac{3}{5} \times \frac{3}{5} + \frac{2}{5} \times \frac{2}{5} = \frac{9}{25} + \frac{4}{25} = \frac{13}{25}$$

Hence the required percentage of cases in which they are likely to contradict each other

$$= \frac{13}{25} \times 100 = 52\% \quad \text{Answer}$$

44. Two balls are drawn from an urn containing 3 white, 5 red and 2 black balls, one by one without replacement. What is the probability that at least one ball is red? (ISC 2019)

Solution: Required probability $= \frac{^5C_2}{^{10}C_2} + \frac{^5C_1 \times ^5C_1}{^{10}C_2} = \frac{10}{45} + \frac{5\times5}{45} = \frac{35}{45} = \frac{7}{9}$ Answer

45. If events A and B are independent, such that P(A) = $\frac{3}{5}$, P(B) = $\frac{2}{3}$, find P (A∪ B). (ISC 2019)

Solution: $P(A \cup B) = P(A) + P(B) - P(A \cap B) = P(A) + P(B) - P(A).P(B) = \frac{3}{5} + \frac{2}{3} - \frac{3}{5} \times \frac{2}{3} = \frac{13}{15}$ Ans.

46. Bag A contains 4 white balls and 3 black balls, while Bag B contains 3 white balls and 5 black balls. Two balls are drawn from Bag A and placed in Bag B. Then what is the probability of drawing a white ball from Bag B? (ISC 2019)

Solution: First two balls are drawn from bag A. Following three cases arise:

Case 1: two balls are white.

Case 2: two balls are black.

Case 3: one ball is white and other black.

$$\text{Required probability} = \frac{^4C_2}{^7C_2} \times \frac{5}{10} + \frac{^3C_2}{^7C_2} \times \frac{3}{10} + \frac{^4C_1 \times ^3C_1}{^7C_2} \times \frac{4}{10}$$

$$= \frac{6}{21} \times \frac{1}{2} + \frac{3}{21} \times \frac{3}{10} + \frac{4\times3}{21} \times \frac{2}{5} = \frac{1}{7} + \frac{3}{70} + \frac{8}{35}$$

$$= \frac{29}{70} \quad \text{Answer}$$

47. A bag contains 5 white, 7 red and 4 black balls. If four balls are drawn one by one with replacement, what is the probability that none is white? (ISC 2020)

Solution: Four balls are drawn one by one with replacement, so each trial is independent.

Probability of not drawing a white ball in first draw $= \frac{11}{16}$.

Probability is same is for 2nd, 3rd and 4th drawing.

$\therefore$ the probability of not drawing a white ball in four drawings $= \frac{11}{16} \times \frac{11}{16} \times \frac{11}{16} \times \frac{11}{16} = \left(\frac{11}{16}\right)^4$ Ans.

48. Let A and B be two events such that

$P(A) = \frac{1}{2}, P(B) = p$ and $P(A \cup B) = \frac{3}{5}$

Find $'p'$ if A and B are independent events. (ISC 2020)

Solution: As A and B are independent events, $P(A \cap B) = P(A).P(B) = \frac{1}{2}\,p$

Now $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{3}{5} = \frac{1}{2} + p - \frac{1}{2}\,p \quad \Rightarrow \frac{3}{5} - \frac{1}{2} = \frac{1}{2}\,p \quad \Rightarrow \frac{1}{10} = \frac{1}{2}\,p \Rightarrow p = \frac{1}{5}$ Answer.

49. Three persons A, B and C shoot to hit a target. Their probabilities of hitting the target are $\frac{5}{6}, \frac{4}{5}$ and $\frac{3}{4}$ respectively. Find the probability that:

(i) exactly two persons hit the target.

(ii) at least one person hit the target. (ISC 2020)

Solution: Let A, B and C be the events that the person A, B and C hit the target respectively.

$\therefore \quad P(A) = \frac{5}{6},\ P(B) = \frac{4}{5}, P(C) = \frac{3}{4}$

$\therefore$ Probability that A does not hit the target $= P(A^c) = 1 - \frac{5}{6} = \frac{1}{6}$ and $P(B^c) = \frac{1}{5}, P(C^c) = \frac{1}{4}$

(i) P (exactly two persons hit the target)

$= P(A \cap B \cap C^c) + P(A \cap B^c \cap C) + P(A^c \cap B \cap C)$

$= P(A).P(B).P(C^c) + P(A).P(B^c).P(C) + P(A^c).P(B).P(C)$

$= \frac{5}{6} \times \frac{4}{5} \times \frac{1}{4} + \frac{5}{6} \times \frac{1}{5} \times \frac{3}{4} + \frac{1}{6} \times \frac{4}{5} \times \frac{3}{4} = \frac{1}{6} + \frac{1}{8} + \frac{1}{10} = \frac{47}{120}$ Answer

(ii) P (none of the three persons hit the target)

$= P(A^c).P(B^c).P(C^c) = \frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} = \frac{1}{120}$

$\therefore$ Probability (at least one person hit the target

$= 1 - P(A^c).P(B^c).P(C^c) = 1 - \frac{1}{120} = \frac{119}{120}$ Answer

Solution

of

Probability (Baye's Theorem)

ISC Class 12 Board Questions

2000 to 2019

1. A firm produces steel pipes in three plants A, B and C with daily production of 500, 1000 and 2000 units respectively. It is known that fractions of defective output produced by the three plants are respectively 0.005, 0.008 and 0.01. A pipe is selected at random from a day's total production and found to be defective. What is the probability that it came from the first plant? (ISC 2005,2000)

Solution: Let E_1, E_2 and E_3 be the event of selecting a pipe produced by three plants A, B and C respectively.

Total no of daily production of pipes = 500 + 1000 + 2000 = 3500

Then $P(E_1) = \frac{500}{3500} = \frac{1}{7}$, $P(E_2) = \frac{1000}{3500} = \frac{2}{7}$, $P(E_3) = \frac{2000}{3500} = \frac{4}{7}$

Let D be the event of selecting a defective pipe.

Then $P(D|E_1) = 0.005$, $P(D|E_2) = 0.008$, $P(D|E_3) = 0.01$

We have to find the probability that the pipe has been produced by plant A, given that it is defective. So, we must find $P(E_1|D)$.

According to Baye's theorem, $P(E_1|D) = \frac{P(E_1).P(D|E_1)}{P(E_1).P(D|E_1)+P(E_2).P(D|E_2)+P(E_3).P(D|E_3)}$

$$= \frac{\frac{1}{7}\times 0.005}{\frac{1}{7}\times 0.005+\frac{2}{7}\times 0.008+\frac{4}{7}\times 0.01} = \frac{\frac{0.005}{7}}{\frac{0.005+0.016+0.04}{7}} = \frac{0.005}{0.061} = \frac{5}{61}$$ Answer

2. An insurance company insured 6000 scooter drivers, 3000 car drivers and 9000 truck drivers. The probability of an accident involving a scooter, a car or a truck is 0.02, 0.06 and 0.3 respectively. One of the insured persons meets with an accident. Find the probability that he is a car driver. (ISC 2002)

Solution: Let E_1, E_2 and E_3 be the events that the insured person is a scooter, a car or a truck driver respectively. Let A be the events that the insured person meets with an accident.

Total no of vehicles = 6000+3000+9000= 18000

So $P(E_1) = \frac{6000}{18000} = \frac{1}{3}$, $P(E_2) = \frac{3000}{18000} = \frac{1}{6}$, $P(E_3) = \frac{9000}{18000} = \frac{1}{2}$

and $P(A|E_1) = 0.02$, $P(A|E_2) = 0.06$, $P(A|E_3) = 0.3$

We have to find the probability that the insured person who meets with an accident is a car driver, i.e., $P(E_2|A)$.

By Bayes' Theorem, $P(E_2|A) = \frac{P(E_2).P(A|E_2)}{P(E_1).P(A|E_1)+P(E_2).P(A|E_2)+P(E_3).P(A|E_3)}$

$$= \frac{\frac{1}{6}\times 0.06}{\frac{1}{3}\times 0.02+\frac{1}{6}\times 0.06+\frac{1}{2}\times 0.3} = \frac{\frac{0.06}{6}}{\frac{0.04+0.06+0.9}{6}} = \frac{0.06}{1} = 0.06$$ Answer

3. A company has two plants which manufacture scooters. Plant **I** manufacture 80% of the scooters while plant **II** manufactures 20% of the scooters. At plant **I**, 85 out of 100 scooters are being of standard quality, while at plant **II** only 65 out of 100 scooters are rated as being of standard quality. If a scooter is of standard quality, what is the probability that it comes from plant **I**? (ISC 2006)

Solution: Let E_1 and E_2be the events that the scooters manufactured by Plant I and Plant II respectively.

$$\therefore \quad P(E_1) = \frac{80}{100} = \frac{4}{5}, \quad P(E_2) = \frac{20}{100} = \frac{1}{5}$$

Let S be the event that the scooter manufactured is of standard quality.

So $P(S|E_1) = \frac{85}{100} = \frac{17}{20}$ and $P(S|E_2) = \frac{65}{100} = \frac{13}{20}$

By Baye's theorem, the required probability that a scooter is of standard quality when it came from Plant I = $P(E_1|S) = \frac{P(E_1).P(S|E_1)}{P(E_1).\ P(S|E_1)+P(E_2).\ P(S|E_2)}$

$$= \frac{\frac{4}{5}\times\frac{17}{20}}{\frac{4}{5}\times\frac{17}{20}+\frac{1}{5}\times\frac{13}{20}} = \frac{68}{68+13} = \frac{68}{81}$$ Answer

4. An insurance company insured 1500 scooter drivers, 2500 car drivers and 4500 truck drivers. The probability of a scooter, a car and a truck meeting with an accident is 0.01, 0.02 and 0.04 respectively. If one of the insured persons meets with an accident, find the probability that he is a scooter driver. (ISC 2007)

Solution: E_1, E_2 and E_3 be the events that the insured person is a scooter driver, a car driver, and a truck driver respectively.

$$\therefore \quad P(E_1) = \frac{1500}{1500+2500+4500} = \frac{1500}{8500} = \frac{3}{17}, \quad P(E_2) = \frac{2500}{8500} = \frac{5}{17}, \quad P(E_3) = \frac{4500}{8500} = \frac{9}{17}$$

Let A be the event that the insured person meets with an accident.

$$\therefore \quad P(A|E_1) = 0.01, \quad P(A|E_2) = 0.02, \quad P(A|E_3) = 0.04$$

We have to find the probability that the insured person who meets with an accident is a scooter driver, i.e. $P(E_1|A)$.

By Baye's Theorem, $P(E_1|A) = \frac{P(E_1).P(A|E_1)}{P(E_1).\ P(A|E_1)+P(E_2).P(A|E_2)+P(E_3).P(A|E_3)}$

$$= \frac{\frac{3}{17}\times 0.01}{\frac{3}{17}\times 0.01+\frac{5}{17}\times 0.02+\frac{9}{17}\times 0.04} = \frac{0.03}{0.03+0.10+0.36} = \frac{0.03}{0.49} = \frac{3}{49}$$ Answer

5. A class consists of 50 students out of which there are 10 girls. In the class 2 girls and 5 boys are rank holders in an examination. If a student is selected at random from the class and is found to be rank holder, what is the probability that the student selected is a girl?

(ISC 2008)

Solution: Let E_1, E_2 be the events that the student is a girl and boy respectively.

Let R be the event that the student is a rank holder.

$\therefore \quad P(E_1) = \frac{10}{50} = \frac{1}{5}, \quad P(E_2) = 1 - P(E_1) = 1 - \frac{1}{5} = \frac{4}{5}$

And $P(R|E_1) = \frac{2}{10} = \frac{1}{5}, \quad P(R|E_2) = \frac{5}{40} = \frac{1}{8}$

We have to find the probability that the student is a rank holder, and she is a girl, i.e., $P(E_1|R)$.

By Baye's Theorem, $P(E_1|R) = \frac{P(E_1).P(R|E_1)}{P(E_1).P(R|E_1)+P(E_2).P(R|E_2)}$

$= \frac{\frac{1}{5}\times\frac{1}{5}}{\frac{1}{5}\times\frac{1}{5}+\frac{4}{5}\times\frac{1}{8}} = \frac{\frac{1}{5}}{\frac{1}{5}+\frac{1}{2}} = \frac{\frac{1}{5}}{\frac{7}{10}} = \frac{1}{5}\times\frac{10}{7} = \frac{2}{7}$ Answer

6. An insurance company insured 4000 doctors, 8000 teachers and 12000 engineers. The probabilities of a doctor, a teacher and an engineer dying before the age of 58 years are 0.01, 0.03 and 0.05 respectively. If one of the insured persons dies before the age of 58 years, find the probability that he is a doctor. (ISC 2009)

Solution: Let E_1, E_2 and E_3 be the events that the insured person is a doctor, teacher and an engineer respectively.

Let A be the event that the insured person dies before 58.

Now $P(E_1) = \frac{4000}{4000+8000+12000} = \frac{4000}{24000} = \frac{1}{6}, P(E_2) = \frac{8000}{24000} = \frac{1}{3}, \quad P(E_3) = \frac{12000}{24000} = \frac{1}{2}$

And $P(A|E_1) = 0.01, \quad P(A|E_2) = 0.03, \quad P(A|E_3) = 0.05$

We have to find the probability that the insured person dies before the age of 58 years and he is a doctor, i.e., $P(E_1|A)$.

By Baye's Theorem, $P(E_1|A) = \frac{P(E_1) \,.\, P(A|E_1)}{P(E_1).P(A|E_1)+P(E_2).P(A|E_2)+P(E_3).P(A|E_3)}$

$= \frac{\frac{1}{6}\times 0.01}{\frac{1}{6}\times 0.01+\frac{1}{3}\times 0.03+\frac{1}{2}\times 0.05} = \frac{\frac{0.01}{6}}{\frac{0.01+0.06+0.15}{6}} = \frac{0.01}{0.22} = \frac{1}{22}$ Answer

7. A factory has three machines A, B and C producing 1500, 2500 and 3000 bulbs per day respectively. Machine A produces 1.5% defective bulbs, machine B produces 2% defective bulbs and machine C produces 2.5% defective bulbs. At the end of the day, a bulb is drawn at random and is found to be defective. What is the probability that this defective bulb has been produced by machine B? (ISC 2010)

Solution: Let E_1, E_2 and E_3 be the events of drawing a bulb produced by machine A, B and C respectively and D the event of drawing a defective bulb.

$\therefore \quad P(E_1) = \frac{1500}{1500+2500+3000} = \frac{1500}{7000} = \frac{3}{14}, \quad P(E_2) = \frac{2500}{7000} = \frac{5}{14}, P(E_3) = \frac{3000}{7000} = \frac{3}{7}$

So $P(D|E_1) = \frac{1.5}{100}, \; P(D|E_2) = \frac{2}{100}, \; P(D|E_3) = \frac{2.5}{100}$

By Baye's Theorem, P (defective bulb is produced by machine B)

$$= P(E_2|D) = \frac{P(E_2)\,.\,P(D|E_2)}{P(E_1).P(D|E_1)+P(E_2).P(D|E_2)+P(E_3).P(D|E_3)}$$

$$= \frac{\frac{5}{14}\times\frac{2}{100}}{\frac{3}{14}\times\frac{1.5}{100}+\frac{5}{14}\times\frac{2}{100}+\frac{3}{7}\times\frac{2.5}{100}} = \frac{\frac{10}{1400}}{\frac{4.5+10+15}{1400}} = \frac{10}{29.5} = \frac{100}{295} = \frac{20}{59}$$ Answer

8. Bag A contains 2 white, 1 black and and 3 red balls. Bag B contains 3 white, 2 black and 4 red balls and bag C contains 4 white, 3 black and 2 red balls. One bag is chosen at random, and 2 balls are drawn at random from that bag. If the randomly drawn balls happen to be red and black, what is the probability that both balls come from bag B? (ISC 2011)

Solution: Let E_1, E_2 and E_3 be the events of drawing bag A and bag B and bag C respectively.

$\therefore\ P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

Let D be the event of drawing a red and a black ball.

$\therefore\ P(D|E_1) = \frac{{}^3C_1 \times {}^1C_1}{{}^6C_2} = \frac{3\times1}{15} = \frac{1}{5},\ P(D|E_2) = \frac{{}^4C_1\times{}^2C_1}{{}^9C_2} = \frac{4\times2}{36} = \frac{2}{9},$

$P(D|E_3) = \frac{{}^2C_1\times{}^3C_1}{{}^9C_2} = \frac{2\times3}{36} = \frac{1}{6}$

By Baye's Theorem,

Probability (drawn balls happen to be red and both come from bag B)

$$= P(E_2|D) = \frac{P(E_2)\,.\,P(D|E_2)}{P(E_1).P(D|E_1)+P(E_2).P(D|E_2)+P(E_3).P(D|E_3)}$$

$$= \frac{\frac{1}{3}\times\frac{2}{9}}{\frac{1}{3}\times\frac{1}{5}+\frac{1}{3}\times\frac{2}{9}+\frac{1}{3}\times\frac{1}{6}} = \frac{\frac{2}{9}}{\frac{1}{5}+\frac{2}{9}+\frac{1}{6}} = \frac{\frac{2}{9}}{\frac{18+20+15}{90}} = \frac{2}{9}\times\frac{90}{53} = \frac{20}{53}$$ Answer

9. In a class of 75 students, 15 are above average, 45 are average and the rest below average achievers. The probability that an above average achieving student fails is 0.005, that an average achieving student fails is 0.05 and the probability of a below average achieving student failing is 0.15. If a student is known to have passed, what is the probability that he is a below average achiever? (ISC 2012)

Solution: Let E_1, E_2 and E_3 be the events that the student selected is above average, average and below average respectively.

$\therefore\ P(E_1) = \frac{15}{75} = \frac{1}{5},\ P(E_2) = \frac{45}{75} = \frac{3}{5},\ P(E_3) = \frac{75-(15+45)}{75} = \frac{1}{5}$

Let S be the event that a student is known to have passed.

$\therefore\ P(S|E_1) = 1 - 0.005 = 0.995,\ P(S|E_2) = 1 - 0.05 = 0.95,$

and $P(S|E_3) = 1 - 0.15 = 0.85$

We have to find probability that a student passes, given that he is a below average achiever, i.e., $P(E_3|S)$.

By Baye's Theorem, $P(E_3|S) = \frac{P(E_3)\,.\,P(S|E_3)}{P(E_1).P(S|E_1)+P(E_2).\,P(S|E_2)+P(E_3).\,P(S|E_3)}$

$= \frac{\frac{1}{5}\times 0.85}{\frac{1}{5}\times 0.995+\frac{3}{5}\times 0.95+\frac{1}{5}\times 0.85} = \frac{0.85}{0.995+2.85+0.85} = \frac{0.85}{4.695} = \frac{170}{939}$ Answer

10. For A, B, and C, the chances of being selected as the manager of a firm are 4 : 1 : 2, respectively. The probabilities for them to introduce a radical change in the marketing strategy are 0.3, 0.8 and 0.5 respectively. If a change takes place; find the probability that it is due to the appointment of B. (ISC 2013)

Solution: Let E_1, E_2 and E_3 be the events that A, B and C is selected as a manager respectively.

$\therefore \quad P(E_1) = \frac{4}{7},\ P(E_2) = \frac{1}{7},\ P(E_3) = \frac{2}{7}$

Let M be the event that radical change occurs in marketing strategy.

$\therefore \quad P(M|E_1) = 0.3, \quad P(M|E_2) = 0.8, \quad P(M|E_3) = 0.5$

We want to find the probability that the radical change in marketing strategy has occurred due to the appointment of B, i.e., $P(E_2|M)$.

By Baye's Theorem,

$$P(E_2|B) = \frac{P(E_2)\,.\,P(M|E_2)}{P(E_1).P(M|E_1)+P(E_2).\,P(M|E_2)+P(E_3).P(M|E_3)}$$

$= \frac{\frac{1}{7}\times 0.8}{\frac{4}{7}\times 0.3+\frac{1}{7}\times 0.8+\frac{2}{7}\times 0.5} = \frac{0.8}{1.2+0.8+1.0} = \frac{0.8}{3} = \frac{4}{15}$ Answer

11. In a bolt factory, three machines A, B and C manufacture 25%, 35% and 40% of the total production respectively and of their respective outputs, 5%, 4% and 2% are defective. A bolt is drawn at random from the total production and it is found to be defective. Find the probability that it was manufactured by machine C. (ISC 2014)

Solution: Let E_1, E_2 and E_3 be the events that the bolt drawn at random is manufactured by machines A, B and C respectively and D be the event that the bolt drawn is defective.

Then probability that the bolt drawn was manufactured by A = $P(E_1) = \frac{25}{100} = \frac{1}{4}$,

$$P(E_2) = \frac{35}{100} = \frac{7}{20} \text{ and } P(E_3) = \frac{40}{100} = \frac{2}{5}$$

and $P(D|E_1) = \frac{5}{100} = \frac{1}{20},\ P(D|E_2) = \frac{4}{100} = \frac{1}{25},\ P(D|E_3) = \frac{2}{100} = \frac{1}{50}$

The required probability that the defective bolt is manufactured by machine C = $P(E_3|M)$.

By Baye's Theorem,

$$P(E_3|D) = \frac{P(E_3).P(D|E_3)}{P(E_1).P(D|E_1)+P(E_2).P(D|E_2)+P(E_3).P(D|E_3)}$$

$= \frac{\frac{2}{5}\times\frac{1}{50}}{\frac{1}{4}\times\frac{1}{20}+\frac{7}{20}\times\frac{1}{25}+\frac{2}{5}\times\frac{1}{50}} = \frac{\frac{1}{125}}{\frac{1}{80}+\frac{7}{500}+\frac{1}{125}} = \frac{16}{69}$ Answer

12. Box I contain two white and three black balls and box II contains four white and one black balls and box III contains three white and four black balls. A dice having three red, two yellow and one green face, is thrown to select the box. If red face turns up, we pick up box I, if a yellow face turns up, we pick up box II, otherwise we pick up box III. Then we draw a ball from the selected box. If the ball drawn is white, what is the probability that the dice had turned up with a red face? (ISC 2015)

Solution: Let E_1, E_2 and E_3 be the events that the box I, box II and box III is selected respectively, and W be the event that the ball drawn from the selected box is white.

$\therefore$ $P(E_1)$ = P (die turns up with a red face) $= \frac{3}{6} = \frac{1}{2}$

$P(E_2) =$ P (die turns up with a yellow face) $= \frac{2}{6} = \frac{1}{3}$

$P(E_3) =$ P (die turns up with a green face) $= \frac{1}{6}$

$P(W|E_1) =$ P (drawing a white ball from box I) $= \frac{2}{5}$

$P(W|E_2) =$ P (drawing a white ball from box II) $= \frac{4}{5}$

$P(W|E_3) =$ P (drawing a white ball from box III) $= \frac{3}{7}$

Required probability is $P(E_1|W)$

By Baye's Theorem, $P(E_1|W) = \frac{P(E_1).\ P(W|E_1)}{P(E_1).\ P(W|E_1)+P(E_2).\ P(W|E_2)+P(E_3).P(W|E_3)}$

$$= \frac{\frac{1}{2}\times\frac{2}{5}}{\frac{1}{2}\times\frac{2}{5}+\frac{1}{3}\times\frac{4}{5}+\frac{1}{6}\times\frac{3}{7}} = \frac{\frac{1}{5}}{\frac{1}{5}+\frac{4}{15}+\frac{1}{14}} = \frac{\frac{1}{5}}{\frac{42+56+15}{210}} = \frac{1}{5}\times\frac{210}{113} = \frac{42}{113}$$ Answer

13. In an automobile factory, certain parts are to be fixed into the chassis in a section before it moves into another section. On a given day, one of the three persons A, B and C carries out this task. A has 45% chance, B has 35% chance and C has 20% chance of doing the task. The probability that A, B and C will take more than the allotted time is $\frac{1}{6}, \frac{1}{10}$ and $\frac{1}{20}$ respectively. If it is found that the time taken is more than the allotted time, what is the probability that A has done the task? (ISC 2016)

Solution: Let E_1, E_2 and E_3 be the events that A, B and C carry out the task respectively. Let M be the event that person has taken more than allotted time.

$\therefore$ $P(E_1) = \frac{45}{100} = \frac{9}{20}$, $P(E_2) = \frac{35}{100} = \frac{7}{20}$, $P(E_3) = \frac{20}{100} = \frac{1}{5}$

and $P(M|E_1) = \frac{1}{6}$, $P(M|E_2) = \frac{1}{10}$, $P(M|E_3) = \frac{1}{20}$

We have to find the probability that if person has taken more time, A has done the task, i.e., we must find $P(E_1|M)$.

By Baye's Theorem, $P(E_1|M) = \frac{P(E_1)\ .\ P(M|E_1)}{P(E_1).P(M|E_1)+P(E_2).P(M|E_2)+P(E_3).P(M|E_3)}$

$$= \frac{\frac{9}{20}\times\frac{1}{6}}{\frac{9}{20}\times\frac{1}{6}+\frac{7}{20}\times\frac{1}{10}+\frac{1}{5}\times\frac{1}{20}} = \frac{\frac{3}{40}}{\frac{3}{40}+\frac{7}{200}+\frac{1}{100}} = \frac{\frac{3}{40}}{\frac{15+7+2}{200}} = \frac{3}{40}\times\frac{200}{24} = \frac{5}{8} = 0.625$$ Answer

14. A fair die is rolled. If face one turns up, a ball is drawn from Bag A. If face 2 or 3 turns up, a ball is drawn from Bag B. If face 4 or 5 or 6 turns up, a ball is drawn from Bag C. Bag A contains 3 red and 2 white balls, Bag B contains 3 red, and 4 white balls and Bag C contains 4 red and 5 white balls. The die is rolled, a bag is picked up and a ball is drawn. If the drawn ball is red; what is the probability that it is drawn from Bag B? (ISC 2017)

Solution: Let E_1, E_2 and E_3 be the events that the Bag A, bag B and Bag C is chosen respectively, and R be the event that the ball drawn is red.

When a dice is rolled, P (getting face 1) $= \frac{1}{6}$, $\therefore P(E_1) = \frac{1}{6}$

P (getting face 2 or 3) $= \frac{2}{6} = \frac{1}{3}$, $\therefore P(E_2) = \frac{1}{3}$

P (getting 4, 5 or 6) $= \frac{3}{6} = \frac{1}{2}$, $\therefore P(E_3) = \frac{1}{2}$

$P(R|E_1) = \frac{3}{3+2} = \frac{3}{5}$, $P(R|E_2) = \frac{3}{3+4} = \frac{3}{7}$, $P(R|E_3) = \frac{4}{4+5} = \frac{4}{9}$

Required probability = P (drawn red ball is from bag B)

$$= P(E_2|R) = \frac{P(E_2).\ P(R|E_2)}{P(E_1).\ P(R|E_1)+P(E_2).P(R|E_2)+P(E_3).\ P(R|E_3)}$$

$$= \frac{\frac{1}{3}\times\frac{3}{7}}{\frac{1}{6}\times\frac{3}{5}+\frac{1}{3}\times\frac{3}{7}+\frac{1}{2}\times\frac{4}{9}} = \frac{\frac{1}{7}}{\frac{1}{10}+\frac{1}{7}+\frac{2}{9}} = \frac{\frac{1}{7}}{\frac{63+90+140}{630}} = \frac{1}{7}\times\frac{630}{293} = \frac{90}{293}$$ Answer

15. Given three identical boxes A, B and C. Box A contains 2 gold and 1 silver coin, Box B contains 1 gold, and 2 silver coins and Box C contains 3 silver coins. A person chooses a box at random and takes out a coin. If the coin drawn is of silver, find the probability that it has been drawn from the Box which has the remaining two coins also of silver. (ISC 2019)

Solution: Let E_1, E_2 and E_3 be the three events that the box A, box B and box C is chosen respectively. Let S be the event that the coin drawn is of silver coin.

$$\therefore\ P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

and $P(S|E_1) = \frac{1}{3}$, $P(S|E_2) = \frac{2}{3}$, $P(S|E_3) = \frac{3}{3} = 1$

We have to find the probability of drawing silver coin from box C , i.e., we have to find $P(E_3|S)$.

By Baye's Theorem, $P\ (E_3|S) = \frac{P(E_3).\ P(S|E_3)}{P(E_1).\ P(S|E_1)+P(E_2).P(S|E_2)+P(E_3).P(S|E_3)}$

$$= \frac{\frac{1}{3}\times 1}{\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{3}+\frac{1}{3}\times 1} = \frac{1}{\frac{1}{3}+\frac{2}{3}+1} = \frac{1}{\frac{6}{3}} = \frac{1}{2}$$ Answer

Solutions of ISC Sem 2 – 2022 Questions
(Baye's Theorem)

16. An insurance company insured 1000 scooter drivers, 2000 car drivers and 4000 truck drivers. The probability of accidents by scooter, car and truck drivers are 0.02, 0.05 and 0.03 respectively. If one of the insured persons meets with an accident, find the probability that he is a truck driver. (ISC Sem 2- 2022)

Solution: Let E_1, E_2, E_3 be the events that insured person is a scooter, car and truck driver respectively. Let A be the event that an insured person meets with the accident. Then

$P(E_1) = \frac{1000}{1000+2000+4000} = \frac{1000}{7000} = \frac{1}{7}$, $P(E_2) = \frac{2000}{7000} = \frac{2}{7}$,

$P(E_3) = \frac{4000}{7000} = \frac{4}{7}$

$P(A/E_1) = 0{\cdot}02$, $P(A/E_2) = 0{\cdot}05$, $P(A/E_3) = 0{\cdot}03$

We have to find the probability that the insured person who meets with an accident is a truck driver i.e., $P(E_3/A)$.

By Bayes' Theorem,

$$P(E_3/A) = \frac{P(E_3).P(A/E_3)}{P(E_1).P(A/E_1)+P(E_2).P(A/E_2)+P(E_3).P(A/E_3)}$$

$$= \frac{\frac{4}{7}\times 0{\cdot}03}{\frac{1}{7}\times 0{\cdot}02+\frac{2}{7}\times 0{\cdot}05+\frac{4}{7}\times 0{\cdot}03} = \frac{4\times 0{\cdot}03}{0{\cdot}02+2\times 0{\cdot}03+4\times 0{\cdot}03} = \frac{0{\cdot}12}{0{\cdot}24} = \frac{1}{2}$$

Probability

Probability distribution and Binomial distribution

ISC Class 12 Board Questions

With Solutions

1992 to 2020

1. In a large collection of bulbs, 3 out of 5 are daffodils and the rest are tulips. If they are planted at random, calculate the probability that in a row of five plants
   (i) all are daffodils.
   (ii) at least four are daffodils. (ISC 1992)

   Solution: Let the bulb planted being daffodils be a success.

   $\therefore p =$ probability of a success = $\frac{3}{5}$ and $q = 1 - p = 1 - \frac{3}{5} = \frac{2}{5}$ and n=5

   $P(r) = {}^{n}C_r \; p^r \; q^{n-r} = {}^{5}C_r \left(\frac{3}{5}\right)^r \left(\frac{2}{5}\right)^{5-r}$

   (i) Since all are daffodils, r = 5

   So, probability that all are daffodils = P (5) = ${}^{5}C_5 \left(\frac{3}{5}\right)^5 \left(\frac{2}{5}\right)^{5-5} = \left(\frac{3}{5}\right)^5 = \frac{243}{3125}$

   (ii) P (at least four are daffodils) = P (4) + P (5)

   $= {}^{5}C_4 \left(\frac{3}{5}\right)^4 \left(\frac{2}{5}\right)^{5-4} + P(5) = \frac{162}{625} + \frac{243}{3125} = \frac{1053}{3125}$

2. Mohan and Sohan play 12 games of chess. Mohan wins 6 games and Sohan 4 games. 2 games end in a draw. They agree to play three more games. calculate the probability that out of these 3 games, 2 games end in a draw. (ISC 1993)

   Solution: Let a game ending in a draw be a success.

   $\therefore p =$ the probability of a success = $\frac{2}{12} = \frac{1}{6}$, $q = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$

   $n =$ no. of games played = 3. r = no. of successes = 2

   $P(r) = {}^{n}C_r \; p^r \; q^{n-r} = {}^{3}C_r \left(\frac{1}{6}\right)^r \left(\frac{5}{6}\right)^{3-r}$

   Hence the required probability of two successes (i.e., two games end in a draw)

   $= P(2) = {}^{3}C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{3-2} = \frac{5}{72}$

3. On an average, out of 12 games of chess played by A and B, A wins 6, B wins 4 and 2 games end in a tie. A and B play tournament of three games. Calculate the probability that

(i) at least two games end in a tie.

(ii) A and B win alternate games, no games end in a tie. (ISC 1994)

Solution: The probability that A wins = $\frac{6}{12} = \frac{1}{2}$ and the probability that B wins = $\frac{4}{12} = \frac{1}{3}$

The probability that the games end in a tie = $\frac{2}{12} = \frac{1}{6}$

(i) p (the probability of success i.e., games end in a tie) = $\frac{1}{6}$ and q = $1 - \frac{1}{6} = \frac{5}{6}$

and n = 3 We have P(r) = ${}^{n}C_r\, p^r\, q^{n-r} = {}^{3}C_r \left(\frac{1}{6}\right)^r \left(\frac{5}{6}\right)^{3-r}$ Here r = 2, 3

Hence the required probability that at least 2 games end in a tie

= P (2) + P (3) = ${}^{3}C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{3-2} + {}^{3}C_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^{3-3} = \frac{15}{216} + \frac{1}{216} = \frac{2}{27}$

(ii) A and B wins alternate games:

If either A wins the first game, B the second and A the third **or** B wins the first game, A the second and B the third.

Hence the required probability = $\frac{1}{2} \times \frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{2} \times \frac{1}{3} = \frac{1}{12} + \frac{1}{18} = \frac{5}{36}$

4. The probability density function y of a continuous variable x is given by $y = \frac{x}{k}, 0 \le x \le 2$ and $y = 0$ for all other values of x. Calculate the values of k and the probability that $x < 1$. (ISC 1994)

Solution: If y is a probability density function of a continuous variable x, then the integral of y between the limits of values of x where $y \neq 0$ is equal to 1.

$$\therefore\ 1 = \int_0^2 \frac{x}{k}.dx \ \Rightarrow 1 = \left[\frac{x^2}{2\,k}\right]_0^2 \Rightarrow 1 = \frac{4}{2\,k} - \frac{0}{2\,k} = \frac{2}{k} \Rightarrow k = 2$$

$$\therefore\ y = \frac{x}{2}$$

We must find the probability so that $0 \le x \le 1$

Required probability $(0 \le x \le 1) = \int_0^1 y\ dx = \int_0^1 \frac{x}{2} dx = \left[\frac{x^2}{4}\right]_0^1 = \frac{1}{4} - 0 = \frac{1}{4}$

5. A pair of dice is thrown five times. If getting a doublet is considered a success, find the probability of 2 successes. (ISC 1995)

Solution: The doublets are (1,1), (2,2), (3,3), (4,4), (5,5), (6,6)

∴ The probability of throwing a doublet = $\frac{6}{36} = \frac{1}{6}$

$\therefore p = \frac{1}{6},\ q = 1 - \frac{1}{6} = \frac{5}{6},\ n = 5, r = 2$

P(r) = ${}^{n}C_r\ p^r\ q^{n-r}$ ∴ P (2) = ${}^{5}C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{5-2} = \frac{625}{3888}$

6. Assume that on an average, one telephone out of 10 is busy, six telephone numbers are randomly selected and called. Find the probability that four of them will be busy. (ISC 1997)

Solution: p = the probability of a telephone number being busy = $\frac{1}{10}$

q = the probability of a telephone number not being busy

$$= 1 - p = 1 - \frac{1}{10} = \frac{9}{10}$$

Thus, we have a binomial distribution with $n = 7, p = \frac{1}{10}, q = \frac{9}{10}$

P(r) = ${}^{n}C_r\, p^r\, q^{n-r}$ Here r =4

Required probability that out of 6 telephone numbers randomly selected, 4 of them will be busy = P (4) = ${}^{6}C_4\, p^4\, q^{6-4}$ = ${}^{6}C_4 \left(\frac{1}{10}\right)^4 \left(\frac{9}{10}\right)^2 = \frac{243}{200000} = 0.001215$

7. Assume that half the population are consumers of chocolate, so that the chance of an individual being a consumer is half. If 100 investigators interviewed 10 individuals each to see whether they are consumers, how many investigators would you expect to report that 3 individuals or less are consumers. (ISC 1998)

Solution: $p = \frac{1}{2},\ q = \frac{1}{2},\ n = 10,$

Required probability = ${}^{10}C_7\, p^3\, q^7 + {}^{10}C_8\, p^2\, q^8 + {}^{10}C_9\, p\, q^9 + q^{10}$

$$= {}^{10}C_7 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^7 + {}^{10}C_8 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^8 + {}^{10}C_9 \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^9 + \left(\frac{1}{2}\right)^{10}$$

$$= (120+45+10+1) \left(\frac{1}{2}\right)^{10} = \frac{176}{1024} = 0.172$$

No of investigators will report = 100 ×0.172 = 17.2 = 17 only

8. A and B play a game which A's chance of winning the game is $\frac{3}{5}$. In a series of 6 games, find the probability that A will win at least 4 games. (ISC 1999)

Solution: $p = \frac{3}{5},\ q = 1 - p = 1 - \frac{3}{5} = \frac{2}{5},\ n = 6$

Required probability = P (4) + P (5) + P (6)

$$= {}^{6}C_4 \left(\frac{3}{5}\right)^4 \left(\frac{2}{5}\right)^2 + {}^{6}C_5 \left(\frac{3}{5}\right)^5 \left(\frac{2}{5}\right)^1 + {}^{6}C_5 \left(\frac{3}{5}\right)^6 = \frac{1701}{3125}$$

9. A die is tossed three times. Getting a '3' or a '5' is considered a success. Find the probability of at least two successes. (ISC 2004)

Solution: $p = \frac{2}{6} = \frac{1}{3},\ q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3},\ n = 3$

P(r) = ${}^{n}C_r\, p^r\, q^{n-r} = {}^{3}C_2 \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{3-2}$

The required probability = P (2) + P (3)

$$= {}^{3}C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^{3-2} + {}^{3}C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^{3-3} = \frac{2}{9} + \frac{1}{27} = \frac{7}{27}$$

10. A coin is tossed five times. What is the probability of getting at least three heads? (ISC 2005)

Solution: $p = \frac{1}{2},\ q = \frac{1}{2}$

Probability of getting 3 heads (at least) = P ($r \geq 3$) = P (3) + P (4) + P (5)

$$= {}^{5}C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2 + {}^{5}C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right) + {}^{5}C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^0 = \frac{1}{2}$$

11. Four dice are thrown simultaneously. If the occurrence of an odd number in a single die is considered a success, find the probability of at most 2 successes. (ISC 2006)

Solution: Probability of success = $p = \frac{3}{6} = \frac{1}{2}$, so $q = 1 - \frac{1}{2} = \frac{1}{2}$

We have a binomial distribution with $n = 4, p = \frac{1}{2}, q = \frac{1}{2}$

Hence, P (r) = ${}^{n}C_r\, p^r\, q^{n-r} = {}^{4}C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{4-r} = {}^{4}C_r \left(\frac{1}{2}\right)^4$

$\therefore$ Probability of at most 2 successes = P (0) + P (1) + P (2)

$= ({}^{4}C_0 + {}^{4}C_1 + {}^{4}C_2) \left(\frac{1}{2}\right)^4 = \frac{11}{16}$

12. Assuming that on an average one telephone out of 10 is busy, seven telephone numbers are randomly selected and called. Find the probability that three of them will be busy. (ISC 2007)

Solution: Let p and q be the probability of success and failures.

Let p = telephone number is dialled and busy and q = telephone number is not dialled and not busy. Let X denotes the number of successes in 7 trials.

$p = \frac{1}{10}$, $q = 1 - \frac{1}{10} = \frac{9}{10}$ No of phone calls selected be n = 7

$\therefore$ The required probability = ${}^{n}C_r\, p^r\, q^{n-r}$

$= {}^{7}C_3 \left(\frac{1}{10}\right)^3 \left(\frac{9}{10}\right)^{7-3} = 35 \times \frac{1}{1000} \times \frac{6561}{10000} = 0.023$

13. In a binomial distribution, the sum of its mean and variance is 1.8. Find the probability of two successes if the event was conducted five times. (ISC 2007)

Solution: Event was conducted 5 times, so $n = 5$.

$$np + npq = 1.8 \Rightarrow 5p + 5pq = 1.8 \Rightarrow p + pq = 0.36$$
$$\Rightarrow p(1+q) = 0.36 \Rightarrow (1-q)(1+q) = 0.36$$
$$\Rightarrow 1 - q^2 = 0.36 \Rightarrow q^2 = 0.64 \Rightarrow q = 0.8 \quad \because 0 \le q \le 1$$
$$\therefore p = 0.2$$
$$\therefore p = 0.2,\ q = 0.8,\ n = 5$$

$\therefore$ Probability of two successes = P(2) = ${}^{5}C_2\, (0.2)^2\, (0.8)^{5-2} = 0.2048 = \frac{128}{625}$

14. 8 coins are thrown simultaneously.

(i) Show that the probability of getting at least 6 heads is $\frac{37}{256}$.

(ii) What is the probability of getting at least 3 heads? (ISC 2008)

Solution: Throwing of eight coins simultaneously is the same as throwing of one coin 8 times. The probability of getting a head(success) = $p = \frac{1}{2}$, $\therefore q = 1 - \frac{1}{2} = \frac{1}{2}$

As the coin is thrown 8 times, so there are 8 trials.

Thus, we have a binomial distribution with $p = \frac{1}{2}, q = \frac{1}{2}, n = 8$.

P (r)= ${}^{8}C_r\, p^r\, q^{n-r} = {}^{8}C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{n-r} = {}^{8}C_r \left(\frac{1}{2}\right)^n = {}^{8}C_r \left(\frac{1}{2}\right)^8 = {}^{8}C_r \times \frac{1}{256}$

(i) Probability of getting at least 6 heads = P ($r \ge 6$)

= P (6) + P (7) + P (8) = ${}^{8}C_6 \times \frac{1}{256} + {}^{8}C_7 \times \frac{1}{256} + + {}^{8}C_8 \times \frac{1}{256} = \frac{37}{256}$

(ii) Probability of getting at least 3 heads = P ($r \ge 3$) $= 1 -$ {P (0) + P (1) + P(2)}

= 1 – {P (8) + P (7) + P (6)} [as P(0)= P(8) , P(1)=P(7), P(2) = P(8)]

$= 1 - \frac{37}{256} = \frac{219}{256}$

15. The mean and variance of a binomial distribution are 4 and 2 respectively. Find the probability of at least six successes. (ISC 2009)

Solution: $np = 4$ and $npq = 2$

$$\therefore q = \frac{2}{4} = \frac{1}{2}, p = 1 - q = 1 - \frac{1}{2} = \frac{1}{2}, \text{ and } n = \frac{4}{p} = \frac{4}{\frac{1}{2}} = 8$$

If r denotes the number of successes,

then P $(r) = {}^{n}C_r\, p^r\, q^{n-r} = {}^{8}C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{8-r} = {}^{8}C_r \left(\frac{1}{2}\right)^8$

$\therefore$ The probability of at least 6 successes = P (6) + P (7) + P (8)

$$= {}^{8}C_6 \left(\frac{1}{2}\right)^8 + {}^{8}C_7 \left(\frac{1}{2}\right)^8 + {}^{8}C_8 \left(\frac{1}{2}\right)^8 = \frac{1}{256}(28 + 8 + 1) = \frac{37}{256}$$

16. Five bad eggs are mixed with 10 good ones. If three eggs are drawn one by one with replacement, find the probability distribution of the number of good eggs drawn. (ISC 2010)

Solution: Since the eggs are drawn one by one with replacement, the events are independent.

Total number of eggs = 5+10 = 15, out of which 10 are good.

$\therefore p =$ probability of drawing a good egg = $\frac{10}{15} = \frac{2}{3}$

and probability of drawing a bad egg = $q = 1 - \frac{2}{3} = \frac{1}{3}$

Thus, we have a binomial distribution with $p = \frac{2}{3}, q = \frac{1}{3}$ and $n = 3$.

Let X denotes the number of good eggs drawn, then X can take values 0, 1, 2, 3.

P (0) = ${}^{3}C_0\, q^3 = 1 \times \left(\frac{1}{3}\right)^3 = \frac{1}{27}$

P (1) = ${}^{3}C_1\, pq^2 = 3 \times \frac{2}{3} \times \left(\frac{1}{3}\right)^2 = \frac{2}{9}$

P (2) = ${}^{3}C_2\, p^2 q = 3 \times \left(\frac{2}{3}\right)^2 \times \frac{1}{3} = \frac{4}{9}$

P (3) = ${}^{3}C_3\, p^3 = 1 \times \left(\frac{2}{3}\right)^3 = \frac{8}{27}$

$\therefore$ The required probability distribution is $\begin{pmatrix} 0 & 1 & 2 & 3 \\ \frac{1}{27} & \frac{2}{9} & \frac{4}{9} & \frac{8}{27} \end{pmatrix}$

17. A box contains 4 red and 5 black marbles. Find the probability distribution of the red marbles in a random draw of 3 marbles. Also, find the mean and standard deviation of the distribution. (ISC 2011)

Solution: Let X denote the number of red marbles in a random draw of 3 marbles, then X can take the values 0, 1, 2, 3.

Three marbles can be drawn out of 9 marbles in ${}^{9}C_3$ ways.

P(X = 0) = P(no red marble) = $\frac{{}^{4}C_0 \times {}^{5}C_3}{{}^{9}C_3} = \frac{5}{42}$

P(X = 1) = P(1 red marble and 2 black marbles) = $\frac{{}^{4}C_1 \times {}^{5}C_2}{{}^{9}C_3} = \frac{20}{42} = \frac{10}{21}$

P(X = 2) = P(2 red marbles and 1 black marble) = $\frac{{}^{4}C_2 \times {}^{5}C_1}{{}^{9}C_3} = \frac{15}{42} = \frac{5}{14}$

P(X = 3) = P(3 red marbles) = $\frac{{}^{4}C_3}{{}^{9}C_3} = \frac{4}{84} = \frac{1}{21}$

∴ The required probability distribution is $\begin{pmatrix} 0 & 1 & 2 & 3 \\ \frac{5}{42} & \frac{10}{21} & \frac{5}{14} & \frac{1}{21} \end{pmatrix}$

Mean = $\sum p_i\, x_i = \frac{5}{42} \times 0 + \frac{10}{21} \times 1 + \frac{5}{14} \times 2 + \frac{1}{21} \times 3 = \frac{4}{3}$

Variance = $\sigma^2 = \sum p_i x_i^2 - \mu^2 = \left(\frac{5}{42} \times 0 + \frac{10}{21} \times 1 + \frac{5}{14} \times 4 + \frac{1}{21} \times 9\right) - \left(\frac{4}{3}\right)^2$

$= \frac{7}{3} - \frac{16}{9} = \frac{5}{9}$

∴ Standard deviation = $\sigma = \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{3}$

18. The probability that a bulb produced by a factory will fuse in 100 days of use is 0.05. Find the probability that out of 5 such bulbs, after 100 days of use:

(i) none fuses

(ii) not more than one fuses.

(iii) more than one fuses.

(iv) at least one fuses. (ISC 2012)

Solution: We consider that the bulb will fuse in 100 days as 'success'.

$p = 0.05 \quad \Rightarrow q = 1 - 0.05 = 0.95\,,\ n = 5$

(i) The probability that none will fuse

= P (0) = ${}^5C_0\, p^0\, q^5 = 1 \times (0.05)^0 \times (0.95)^5 = 0.77$

(ii) The probability that not more than one fuses

= P (0) + P (1) = P (0) + ${}^5C_1\ p^1\, q^4$ = 0.77 + 5 $\times (0.05) \times (0.95)^4 = 0.97$

(iii) The probability that more than one fuses

= 1 – {P (0) + P (1)} = 1 – 0.97 = 0.03

(iv) The probability that at least one fuses

= 1 – P (0) = 1 – 0.77 = 0.23

19. If the sum and the product of the mean and variance of a binomial distribution are 1.8 and 0.8 respectively, find the probability distribution and the probability of at least one success. (ISC 2013)

Solution: Let, p be the probability of success and q that of failure in any one trial and n be the number of trials.

$np + npq = 1.8 \quad \Rightarrow np(1+q) = 1.8$ ---------- (i)

$np.npq = 0.8 \quad \Rightarrow\ n^2\, p^2\, q = 0.8$ ---------- (ii)

Dividing the square of (i) by (ii), we get

$\frac{n^2p^2(1+q)^2}{n^2p^2q} = \frac{1.8^2}{0.8} \Rightarrow \frac{(1+q)^2}{q} = \frac{81}{20}$

$\Rightarrow 20\, q^2 - 41\, q + 20 = 0 \quad \Rightarrow\ (5q-4)(4q-5) = 0 \Rightarrow q = \frac{5}{4}, \frac{4}{5}$ but $0 < q < 1$

$\therefore\ q = \frac{4}{5} \quad \Rightarrow\ p = 1 - q = 1 - \frac{4}{5} = \frac{1}{5}$ and $n.\frac{1}{5}\left(1 + \frac{4}{5}\right) = 1.8 \ \Rightarrow n = 5$

Hence, the binomial distribution is $(q+p)^n = \left(\frac{4}{5} + \frac{1}{5}\right)^5$

Probability of at least one success = 1 – P (0) = $1 - q^5 = 1 - \left(\frac{4}{5}\right)^5 = 1 - \frac{1025}{3125} = \frac{2101}{3125}$

20. On dialling certain telephone numbers, assume that on an average one telephone number out of 5 is busy, ten telephone numbers are randomly selected and dialled. Find the probability that at least three of them will be busy. (ISC 2014)

Solution: $p = \frac{1}{5},\ q = 1 - p = 1 - \frac{1}{5} = \frac{4}{5}$

The probability that at least three telephone numbers are busy

= P $(r \geq 3) = 1 - P(r < 3)$

$= 1 - [\ {}^{10}C_0 \left(\frac{1}{5}\right)^0 \left(\frac{4}{5}\right)^{10} + {}^{10}C_1 \left(\frac{1}{5}\right)^1 \left(\frac{4}{5}\right)^9 + {}^{10}C_2 \left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right)^8\ ] = 0.322$

21. Five dice are thrown simultaneously. If the occurrence of an odd number in a single dice is considered a success, find the probability of maximum three successes. (ISC 2015)

Solution: Probability of success = probability of getting an odd number = $p = \frac{3}{6} = \frac{1}{2}$

Probability of not getting an odd number = $q = 1 - \frac{1}{2} = \frac{1}{2}$

Hence, P(r) = ${}^nC_r\, p^r\, q^{n-r} = {}^5C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{5-r}$

Probability of maximum three success= P$(r \leq 3)$ = P(0) + P(1) + P(2) + P(3)

$= {}^5C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^5 + {}^5C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^4 + {}^5C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^3 + {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2$

$= \frac{1}{32} + 5 \times \frac{1}{2} \times \frac{1}{16} + 10 \times \frac{1}{4} \times \frac{1}{8} + 10 \times \frac{1}{8} \times \frac{1}{4} = \frac{13}{16}$

22. The difference between mean and variance of a binomial distribution is 1 and the difference of their squares is 11. Find the distribution. (ISC 2016)

Solution: $np - npq = 1$ ---- (i)

$(np)^2 - (npq)^2 = 11$

$\Rightarrow \quad (np + npq)(np - npq) = 11$

$\Rightarrow \quad np + npq = 11$ ------ (ii)

From (i) and (ii), $2np = 12 \Rightarrow np = 6$ and $2npq = 10 \Rightarrow npq = 5$

$\therefore \quad q = \frac{5}{6}$ and $p = 1 - q = 1 - \frac{5}{6} = \frac{1}{6}$ and $n = \frac{6}{p} = \frac{6}{\frac{1}{6}} = 36$

$\therefore$ The binomial distribution is $(q + p)^n = \left(\frac{5}{6} + \frac{1}{6}\right)^{36}$

23. From a lot of six items containing two defective items, a sample of four items are drawn at random. Let the random variable x denote the number of defective items in the sample. If the sample is drawn without replacement, find:

(a) the probability distribution of x

(b) mean of x

(c) variance of x. (ISC 2018)

Solution: In 6 items 2 defective and 4 non-defective items.

Let P is the probability of defective items

Given $x =$ number of defective items.

(a) Hence, P (x =0) = $\frac{^4C_4}{^6C_4} = \frac{1}{15}$

P (x =1) = $\frac{^2C_1 \times ^4C_3}{^4C_4} = \frac{8}{15}$

P (x =2) = $\frac{^2C_1 \times ^4C_2}{^4C_4} = \frac{6}{15}$

(b) Mean $(\bar{x}) = \sum P_i x_i = \frac{1}{15} \times 0 + \frac{8}{15} \times 1 + \frac{6}{15} \times 2 = \frac{20}{15} = \frac{4}{3}$

(c) Variance = $(\sigma^2) = \sum P_i x_i^2 - (\sum P_i x_i)^2 = \frac{32}{15} - \left(\frac{4}{3}\right)^2 = \frac{16}{45} = 0.35$

24. Determine the binomial distribution where mean is 9 and standard deviation is $\frac{3}{2}$. Also, find the probability of obtaining at most one success. (ISC 2019)

Solution: mean = 9 $\Rightarrow np = 9$ ---------- (i)

S.D = $\frac{3}{2}$ $\sqrt{variance} = \frac{3}{2} \Rightarrow \sqrt{npq} = \frac{3}{2} \Rightarrow npq = \frac{9}{4}$ ------- (ii)

Dividing (ii) by (i) $\frac{npq}{np} = \frac{9}{4} \times \frac{1}{9} \Rightarrow q = \frac{1}{4} \quad \therefore p = 1 - q = 1 - \frac{1}{4} = \frac{3}{4}$

$n.\frac{3}{4} = 9 \Rightarrow n = 12$

Thus, binomial distribution is $\left(\frac{3}{4} + \frac{1}{4}\right)^{12}$

P (at most one success) = P (0) +P (1)

$= {}^{12}C_0 \left(\frac{3}{4}\right)^0 \left(\frac{1}{4}\right)^{12} + {}^{12}C_1 \left(\frac{3}{4}\right)^1 \left(\frac{1}{4}\right)^{11} = \left(\frac{1}{4}\right)^{12} + 36 \left(\frac{1}{4}\right)^{12} = \frac{37}{4^{12}}$

25. The probability that a bulb produced in a factory will fuse after 150 days of use is 0.05. Find the probability that out of five such balls bulbs:

(i) none will fuse after 150 days of use.

(ii) not more than one will fuse after 150 days of use.

(iii) more than one will fuse after 150 days of use.

(iv) at least one will fuse after 150 days of use. (ISC 2020)

Solution: Let p and q be the probability of success and failure.

p = a bulb will fuse in 150 days of use = 0.05 = $\frac{1}{20}$

q = a bulb will not fuse in 150 days of use = $1 - \frac{1}{20} = \frac{19}{20}$

Let X denotes the number of successes in 5 bulbs.

(i) P(None fuse) = P(0) = $^5C_0 \left(\frac{1}{20}\right)^0 \left(\frac{19}{20}\right)^5 = \left(\frac{19}{20}\right)^5 = (0.95)^5$

(ii) P(Not more than one fuse) = P($X \leq 1$) = P(0) + P(1)

$= {}^5C_0 \left(\frac{1}{20}\right)^0 \left(\frac{19}{20}\right)^5 + {}^5C_1 \left(\frac{1}{20}\right)^1 \left(\frac{19}{20}\right)^4 = (0.95)^5 + \frac{1}{4} \times (0.95)^4$

$= (0.95)^4 (0.95 + 0.25) = 1.2 \times (0.95)^4$

(iii) P(More than one fuse) = P($X > 1$) = 1 – P($X \leq 1$) = $1 - 1.2 \times (0.95)^4$

(iv) P(at least one fuse) = P($X \geq 1$) = $1 - P(0) = 1 - (0.95)^5$